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DYNAMICS OF A COUPLED SHELL/FLUID SYSTEM

by

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PREFACE

The work reported herein was performed as part of the base technology activity under the Flow Induced Vibration Programs (189a Nos. 02659 and 02683) sponsored by ERDA/RRD. The overall objective of the activity is to develop new and/or improved analytical methods and guidelines for designing LMFBR components to avoid detrimental flow induced vibration.

In a typical reactor system, many components are susceptible to flow-induced vibration. One set of components includes nominally circular cylindrical shells coupled to other shells through a liquid, such as shrouds, thermal liners and flow directing baffles. Designing to avoid large amplitude motion, that is, to avoid a resonance condition or unstability condition, and the prediction of component response, require knowledge of the dynamic behavior of the components. However, two circular cylindrical shells separated by a narrow fluid gap do not respond as single shells, rather, interaction with the fluid causes coupled vibration. The fundamental natural frequency of the coupled system will be lower than that of a single shell.

For the purpose of understanding the dynamics and controlling the vibration in reactor components, this paper presents a study of two cylindrical shells arranged concentrically and containing and separated by fluid. An exact frequency equation is obtained for the general case and an approximate closed-form solution is given for the shell system with an incompressible fluid. The results illustrate the significance of the interaction of two shells in a liquid and are useful in design and evaluation of system components containing circular cylindrical shells. Of particular importance are the added mass coefficients obtained in the paper. Those coefficients will be used in the development of sets of design curves to be included in a future design guide.

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NOMENCLATURE

a,b,d,e	Constants given by Fq. (19)
c	phase velocity
c_i	Speed of sound in fluid
E_i	Young's modulus
f_j (j=1,2,3,4,5)	Coefficients given by Eq. (16)
$F_{in}(r)$, $G_{in}(r)$	Functions given by Eq. (11)
h_i	Thickness of shell
ℓ	Axial wavelength
n	Circumferential wave number
P_i	Fluid pressure
q_i	Radial surface loading component
r	Radial coordinate
R_i	Radius of shell
t	Time
u_i, $\bar{u}_i$	Axial displacement of shell
v_i, $\bar{v}_i$	Circumferential displacement of shell
V_i	Fluid radial velocity
w_i, $\bar{w}_i$	Radial displacement of shell
z	Axial coordinate
α_i	$2\pi R_i/\ell$
γ_i	$c_i\left[\frac{\text{[illegible]}(1-\nu_i^2)}{E_i}\right]^{1/2}$
δ_i	h_i/R_i
θ	Angular coordinate
μ_i	$\frac{\sigma_i R_i}{\rho_i h_i}$
ν_i	Poisson's ratio

NOMENCLATURE (cont'd)

ρ_i	Shell density
σ_i	Fluid density
ϕ_i, $\overline{\phi}_k$	Fluid velocity potential
ω	Circular frequency
Ω_i	$R_i\omega \left[\frac{\rho_i(1-\nu_i^2)}{E_i}\right]^{1/2}$
$\overline{\Omega}_i$	Dimensionless shell frequency in vacuo given by Eq. (19).

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ABSTRACT

This paper presents a study of two concentrically located circular cylindrical shells containing and separated by fluids. An exact frequency equation is derived for the general case and an approximate closed-form solution is obtained for the shell system with an incompressible fluid. It is found that the lowest frequency of the coupled system is associated with one of the out-of-phase modes, and is lower than the frequencies of the individual shells.

I. INTRODUCTION

Flow-induced vibrations are of great concern in the development and design of Liquid-Metal-Cooled Fast Breeder Reactor systems. One set of problems includes nominally circular cylindrical shells coupled to other shells through a liquid. Examples include shrouds, flow directing baffles, and thermal liners. In a typical reactor system such components are subject to various excitation sources including fluid flow and structural borne disturbances. The particular case of two concentric shells containing and coupled through the fluid is treated here. For example, the motion of a thermal liner may be influenced, significantly, by the surrounding fluid and the reactor vessel or other reactor internals. This requires a kind of "subsystem" treatment in place of analyzing the vibration behavior as a single component. This study of a concentric double shell system provides an improved characterization of the vibration behavior for designing components such as those given in the examples.

Many studies have been made on the vibration of cylindrical shells containing fluid. Accounts of the work in this field have been published [1]. The system studied in this paper is characterized through the use of Flügge's shell equations and potential flow theory. An exact frequency equation is presented for the general case and a closed-form solution is given for the shell system with an incompressible fluid.

II. THE FREQUENCY EQUATION

Consider two concentric circular cylindrical shells containing and separated by acoustic media as shown in Fig. 1. The motion of the shells is described by the following Flügge's shell equations [2]

$$
\begin{aligned}
&\left[\frac{\partial^2}{\partial z^2} + \left(\frac{1-\nu_i}{2R_i^2}\right)\left(1 + \frac{h_i^2}{12\,R_i^2}\right)\frac{\partial^2}{\partial \theta^2}\right]u_i + \frac{1+\nu_i}{2\,R_i}\frac{\partial^2 v_i}{\partial z \partial \theta} \\
&\quad + \left[\frac{\nu_i}{R_i}\frac{\partial}{\partial z} - \frac{h_i^2}{12\,R_i}\frac{\partial^3}{\partial z^3} + \frac{(1-\nu_i)h_i^2}{24\,R_i^3}\frac{\partial^3}{\partial z \partial \theta^2}\right]w_i = \frac{\rho_i(1-\nu_i^2)}{E_i}\frac{\partial^2 u_i}{\partial t^2} \\
&\frac{1+\nu_i}{2\,R_i}\frac{\partial^2 u_i}{\partial z \partial \theta} + \left[\frac{1}{R_i^2}\frac{\partial^2}{\partial \theta^2} + \frac{1-\nu_i}{2}\left(1 + \frac{h_i^2}{4\,R_i^2}\right)\frac{\partial^2}{\partial z^2}\right]v_i \\
&\quad + \left[\frac{1}{R_i^2}\frac{\partial}{\partial \theta} - \frac{(3-\nu_i)h_i^2}{24\,R_i^2}\frac{\partial^3}{\partial \theta \partial z^2}\right]w_i = \frac{\rho_i(1-\nu_i^2)}{E_i}\frac{\partial^2 v_i}{\partial t^2} \qquad (1) \\
&\left[-\frac{h_i^2}{12\,R_i}\frac{\partial^3}{\partial z^3} + \frac{\nu_i}{R_i}\frac{\partial}{\partial z} + \frac{(1-\nu_i)h_i^2}{24\,R_i^3}\frac{\partial^3}{\partial z \partial \theta^2}\right]u_i + \left[-\frac{(3-\nu_i)h_i^2}{24\,R_i^2}\frac{\partial^3}{\partial \theta \partial z^2} + \frac{1}{R_i^2}\frac{\partial}{\partial \theta}\right]v_i \\
&\quad + \left[\frac{1}{R_i^2} + \frac{h_i^2}{12\,R_i^4} + \frac{h_i^2}{12}\frac{\partial^4}{\partial z^4} + \frac{h_i^2}{6\,R_i^2}\frac{\partial^4}{\partial z^2 \partial \theta^2} + \frac{h_i^2}{12\,R_i^4}\frac{\partial^4}{\partial \theta^4} + \frac{h_i^2}{6\,R_i^4}\frac{\partial^2}{\partial \theta^2}\right]w_i \\
&\quad = -\frac{\rho_i(1-\nu_i^2)}{E_i}\frac{\partial^2 w_i}{\partial t^2} + \frac{(1-\nu_i^2)}{E_i h_i}q_i
\end{aligned}
$$

where the index i denotes the variables associated with the inner shell (i = 1) and outer shell (i = 2); u_i, v_i and w_i are the displacement components of the shell middle surface; r, θ, and z are cylindrical coordinates; t is the time; and q_i is the radial surface loading components per unit area. The physical characteristics of the shells are defined by the mean radius R_i, wall thickness h_i, density ρ_i, Young's modulus E_i and Poisson's ratio ν_i.

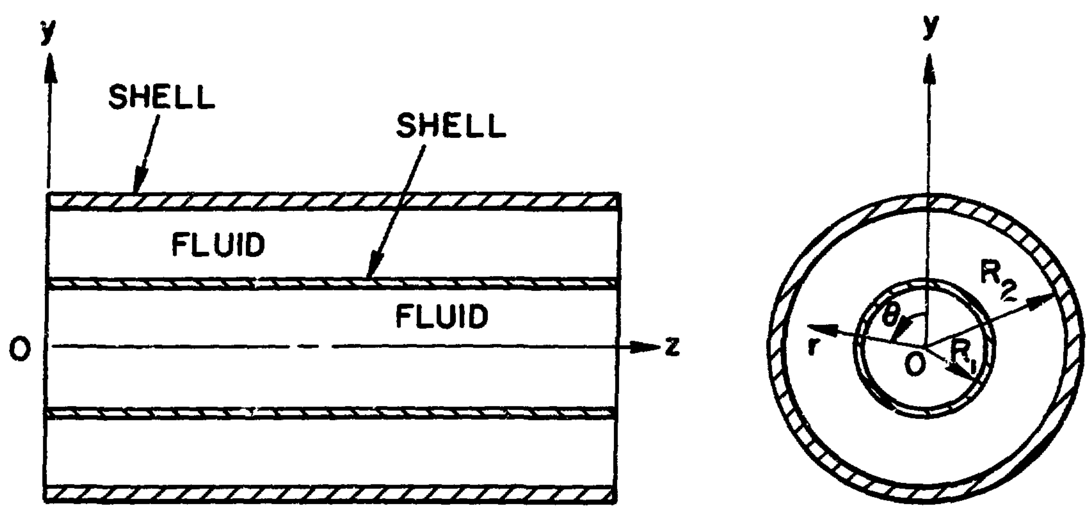

Fig. 1. A Coupled Fluid/Shell System.

The governing fluid field equation can be expressed as

$$\nabla^2 \phi_i = \frac{1}{c_i^2} \frac{\partial^2 \phi_i}{\partial t^2} \tag{2}$$

where the index i denotes the central region (i = 1) and annular region (i = 2) and c_i is the sound velocity. The corresponding fluid radial velocity and fluid pressure are

$$V_i = \frac{\partial \phi_i}{\partial r} \tag{3}$$

$$P_i = -\sigma_i \frac{\partial \phi_i}{\partial t} \tag{4}$$

where σ_i is the fluid density.

The interface conditions are

$$\begin{aligned} V_1\Big|_{r=R_1} &= \frac{\partial w_1}{\partial t} \\ V_2\Big|_{r=R_1} &= \frac{\partial w_1}{\partial t} \\ V_2\Big|_{r=R_2} &= \frac{\partial w_2}{\partial t} \end{aligned} \tag{5}$$

and the surface loading q_i are given by

$$\begin{aligned} q_1 &= P_1\Big|_{r=R_1} - P_2\Big|_{r=R_1} \\ q_2 &= P_2\Big|_{r=R_2} \end{aligned} \tag{6}$$

Solutions of the following form are assumed

$$\begin{aligned} u_i &= \bar{u}_i \cos n\theta \exp[i2\pi(ct - z)/\ell] \\ v_i &= \bar{v}_i \sin n\theta \exp[i2\pi(ct - z)/\ell] \\ w_i &= \bar{w}_i \cos n\theta \exp[i2\pi(ct - z)/\ell] \end{aligned} \tag{7}$$

where n is the circumferential wave number, c is the phase velocity, ℓ is the axial wave length, and $\bar{u}_i$, $\bar{v}_i$, and $\bar{w}_i$ are arbitrary constants to be determined. Similarly, the fluid velocity potential may be defined by the following expression:

$$\phi = \bar{\phi}_i(r)\cos n\theta \exp[i2\pi(ct - z)/\ell] \tag{8}$$

Substituting Eq. (8) into (2) gives the following form of Bessel's equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\,\frac{d\bar{\phi}_i}{dr}\right) - \left\{\frac{n^2}{r^2} + \frac{4\pi^2}{\ell^2}\left(1 - \frac{c}{c_i}\right)^2\right\}\bar{\phi}_i = 0 \tag{9}$$

Integrating Eq. (9) and applying the condition of regularity at r = 0 and the interface conditions at $r = R_1$ and R_2, Eqs. (4), yield

$$\bar{\phi}_1(r) = i2\pi\left(\frac{c}{\ell}\right)\frac{F_{1n}(r)}{F'_{1n}(R_1)}\,\bar{w}_1$$

$$\begin{aligned}\bar{\phi}_2(r) = \frac{i2\pi\,\frac{c}{\ell}}{F'_{2n}(R_1)G'_{2n}(R_2) - F'_{2n}(R_2)G'_{2n}(R_1)} &\{[G'_{2n}(R_2)F_{2n}(r) - F'_{2n}(R_2)G_{2n}(r)]\bar{w}_1 \\ &+ [F'_{2n}(R_1)G_{2n}(r) - G'_{2n}(R_1)F_{2n}(r)]\bar{w}_2\}\end{aligned} \tag{10}$$

where

$$\begin{aligned}F_{in}(r) &= I_n\left[\frac{2\pi}{\ell}\left(1 - \frac{c}{c_i}\right)^{1/2} r\right] \qquad c_i > c \\ &= J_n\left[\frac{2\pi}{\ell}\left(\frac{c}{c_i} - 1\right)^{1/2}\right] \qquad c_i < c \\ G_{in}(r) &= K_n\left[\frac{2\pi}{\ell}\left(1 - \frac{c}{c_i}\right)^{1/2} r\right] \qquad c_i > c \\ &= Y_n\left[\frac{2\pi}{\ell}\left(\frac{c}{c_i} - 1\right)^{1/2} r\right] \qquad c_i < c\end{aligned} \tag{11}$$

and the prime denotes differentiation with respect to r.

Introduce the following dimensionless variables:

$$\alpha_i = \frac{2\pi R_i}{\ell} \qquad \gamma_i = c_i\left[\frac{\rho_i(1-\nu_i^2)}{E_i}\right]^{1/2}$$

$$\delta_i = \frac{h_i}{R_i} \tag{12}$$

$$\mu_i = \frac{\sigma_i R_i}{\rho_i h_i}$$

and the dimensionless frequency Ω_i which is related to the circular frequency of vibration ω by

$$\Omega_i = R_i\omega\left[\frac{\rho_i(1-\nu_i^2)}{E_i}\right]^{1/2} \tag{13}$$

Substituting Eqs. (7) into (1) and using Eqs. (3), (4), (6), (8), and (10) gives six linear, algebraic homogeneous equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{12} & a_{14} & a_{15} & 0 & 0 & 0 \\ a_{13} & a_{15} & a_{16} & 0 & 0 & a_{17} \\ 0 & 0 & 0 & a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 & a_{22} & a_{24} & a_{25} \\ 0 & 0 & a_{27} & a_{23} & a_{25} & a_{26} \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{w}_1 \\ \bar{u}_2 \\ \bar{v}_2 \\ \bar{w}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

where

$$a_{i1} = -\alpha_i^2 - \left(\frac{1-\nu_i}{2}\right)\left(1 + \frac{\delta_i^2}{12}\right) n^2 + \Omega_i^2$$

$$a_{i2} = -\sqrt{-1}\left(\frac{1+\nu_i}{2}\right)\alpha_i n$$

$$a_{i3} = -\sqrt{-1}\left[\nu_i\alpha_i + \frac{\delta_i^2}{12}\alpha_i^3 - \frac{\delta_i^2}{24}(1-\nu_i)\alpha_i n^2\right] \tag{15}$$

$$a_{i4} = n^2 + \left(\frac{1 - \nu_i}{2}\right)\left(1 + \frac{\delta_i^2}{4}\right)\alpha_i^2 - \Omega_i^2$$

$$a_{i5} = n^2 + \frac{\delta_i^2}{24}(3 - \nu_i)n\alpha_i^2$$

$$a_{16} = 1 + \frac{\delta_1^2}{12}[1 - 2n^2 + (\alpha_1^2 + n^2)^2] - \Omega_1^2 - \mu_1 f_1 \Omega_1^2 - \mu_1\left(\frac{\sigma_2}{\sigma_1}\right) f_2\Omega_1^2$$

$$a_{17} = \mu_1\left(\frac{\sigma_2}{\sigma_1}\right) f_5\Omega_1^2 \qquad \text{(15) (contd.)}$$

$$a_{26} = 1 + \frac{\delta_2^2}{12}[1 - 2n^2 + (\alpha_2^2 + n^2)^2] - \Omega_2^2 - \mu_2 f_3\Omega_2^2$$

$$a_{27} = \mu_2 f_4\Omega_2^2$$

and

$$\Delta = F'_{2n}(R_1)G'_{2n}(R_2) - F'_{2n}(R_2)G'_{2n}(R_1)$$

$$f_1 = \frac{F_{1n}(R_1)}{F'_{1n}(R_1)}$$

$$f_2 = \frac{1}{\Delta}[G'_{2n}(R_2)F_{2n}(R_1) - F'_{2n}(R_2)G_{2n}(R_1)]$$

$$f_3 = \frac{1}{\Delta}[F'_{2n}(R_1)G_{2n}(R_2) - G'_{2n}(R_1)F_{2n}(R_2)] \qquad (16)$$

$$f_4 = \frac{1}{\Delta}\left(\frac{R_1}{R_2}\right)[G_{2n}(R_2)F'_{2n}(R_2) - F_{2n}(R_2)G'_{2n}(R_2)]$$

$$f_5 = \frac{1}{\Delta}\left(\frac{R_2}{R_1}\right)[F'_{2n}(R_1)G_{2n}(R_1) - G'_{2n}(R_1)F_{2n}(R_1)]$$

The frequency equation is obtained by setting the determinant of the coefficient matrix in Eq. (14) equal to zero; it can be written as

$$F(\Omega_i, \alpha_i, \delta_i, \mu_i, \nu_i, \gamma_i, \sigma_2/\sigma_1, n) = 0 \qquad (17)$$

Several limiting cases can be deduced from Eq. (14):

(a) For $\sigma_i = 0$, the equation gives the frequency for two empty shells.

(b) For n = 0, the equation gives the dispersion relation for axially symmetric modes.

(c) $\alpha_i = 0$ yields the frequency equation of the circumferential motions.

(d) When either one of the shells is rigid, the equation becomes the frequency equation of the other shell.

III. GENERAL FREQUENCY SPECTRA

For a given shell-fluid system, Eq. (17) can be solved for the frequency. In presentation, a double shell system with sodium at 250°F is considered. Both inner and outer shells are made of stainless steel and both central and annular regions are filled with sodium. The shell and fluid properties are given as follows: $E_1 = E_2 = 27.5 \times 10^6$ psi, $\nu_1 = \nu_2 = 0.27$, $\rho_1 = \rho_2 = 0.2885$ lb/in.^{3}, $\delta_1 = 0.0074$, $\delta_2 = 0.0180$, $\sigma_1 = \sigma_2 = 0.0333$ lb/in.^{3}, $c_1 = c_2 = 102240$ in./sec and $R_2/R_1 = 2$. Equation (17) was explored for the following range of Ω_1 and α_1: $0 \leq \Omega_1 \leq 5.0$ and $0 \leq \alpha_1 \leq 3.0$.

Figures 2, 3, and 4 show the dispersion curves for n = 1 and 2 and for three cases: (a) a shell system in vacuo; (b) a shell system with incompressible fluid; and (c) a shell system with compressible fluid.

In case a, there are six branches; each shell has three dispersion curves and they are independent of each other. The first two branches, in which the motion is predominantly radial motion, are of most importance in practical considerations. In case b, there are also six branches only. In this case, the fluid in the annular region effectively links the two shell motions and the shells are strongly coupled. The lowest two branches are predominantly associated with the radial motions. It is seen that the frequencies of these two branches are lowered significantly. In the first mode, the two shells move out of phase, while in the second mode, the two shells move in phase. In the first mode, since the fluid in the annular region has to be displaced, the fluid inertia's effect is much more important.

In case c, when the fluid is compressible, there exist infinite branches. Some are predominantly associated with structural motion and some are predominantly associated with fluid motion; they may be called structural modes and acoustic modes, respectively. For small μ_i, the interaction between

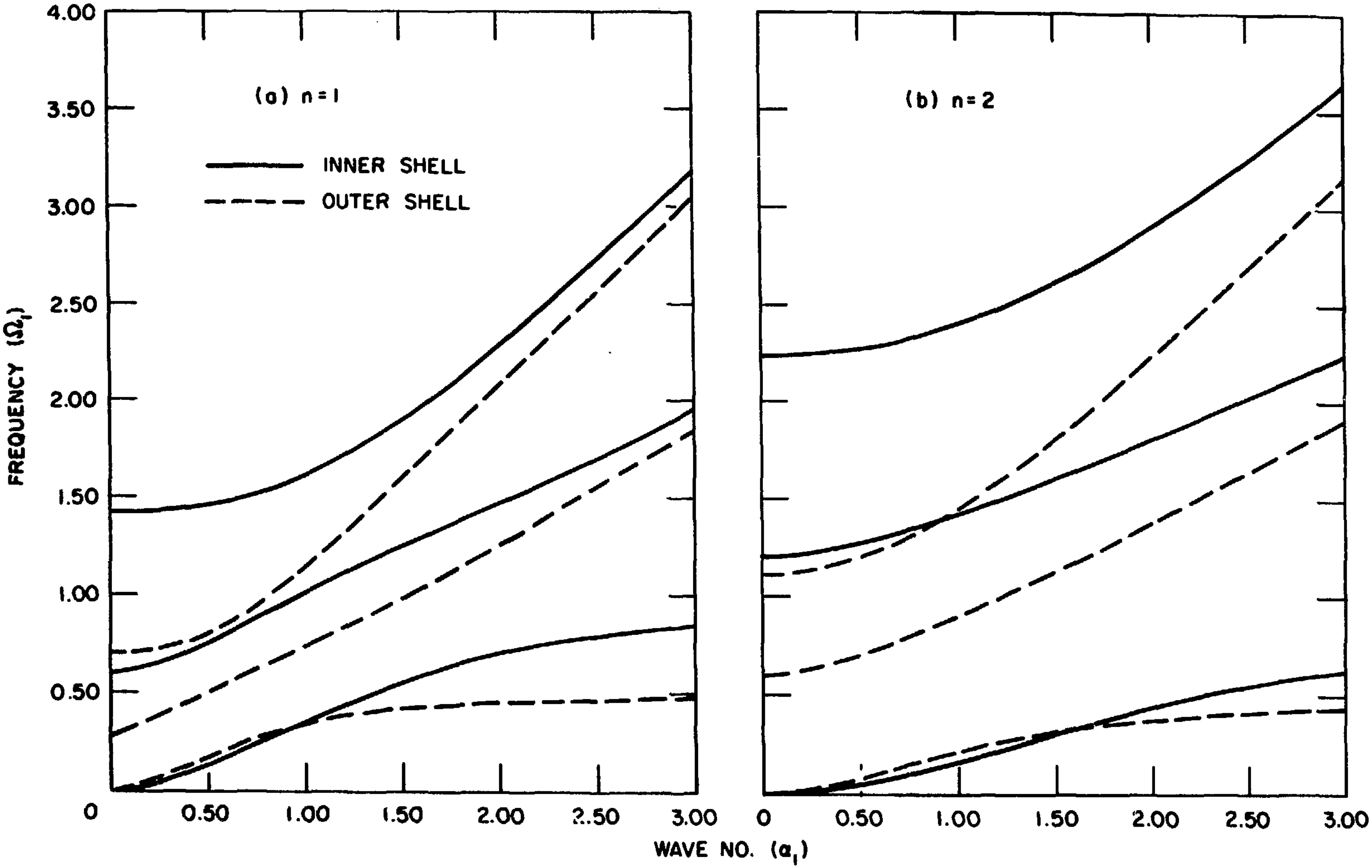

Fig. 2. Frequency Spectra of a Shell System in Vacuo.

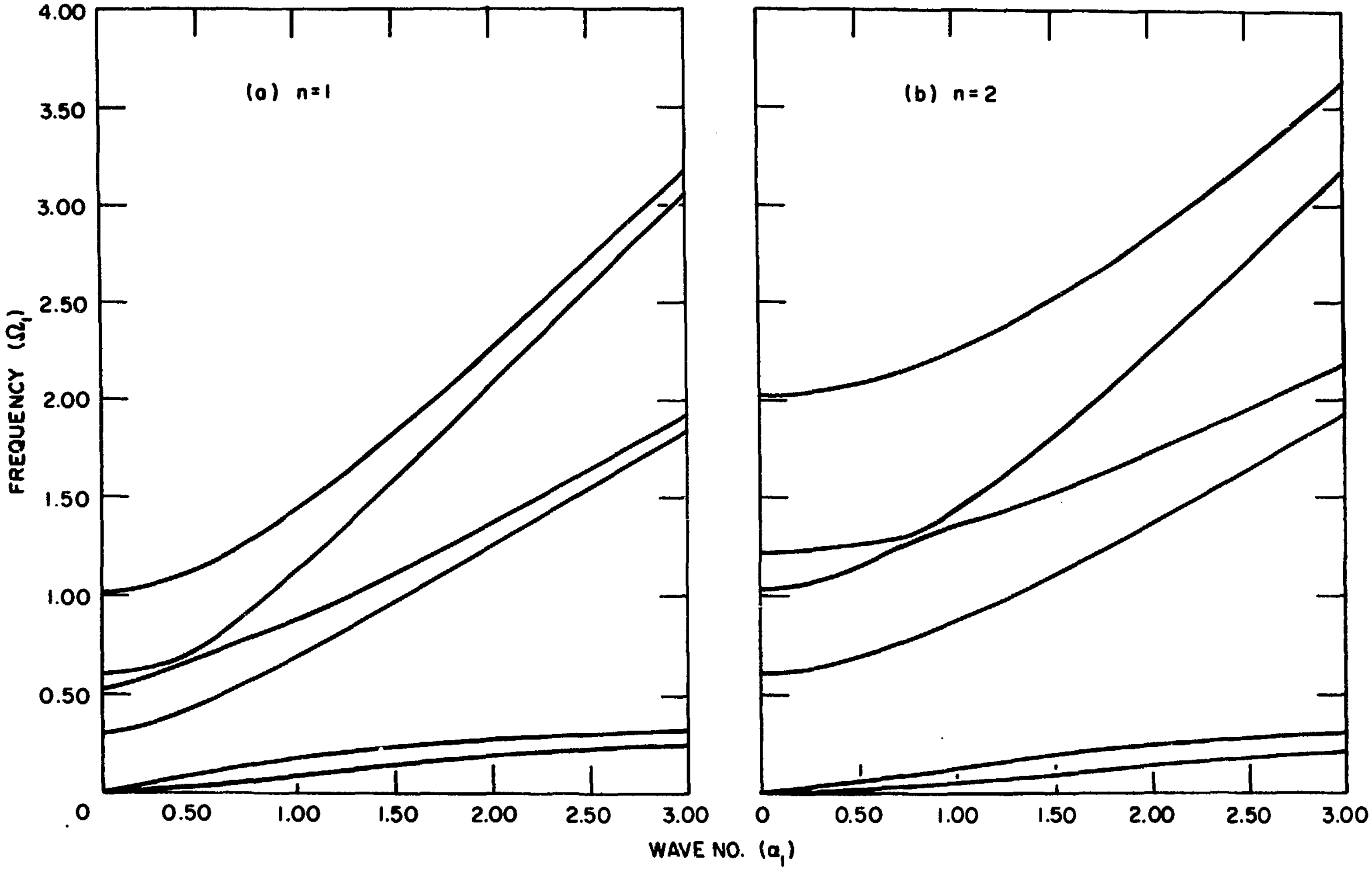

Fig. 3. Frequency Spectra of a Shell System Containing Incompressible Fluid.

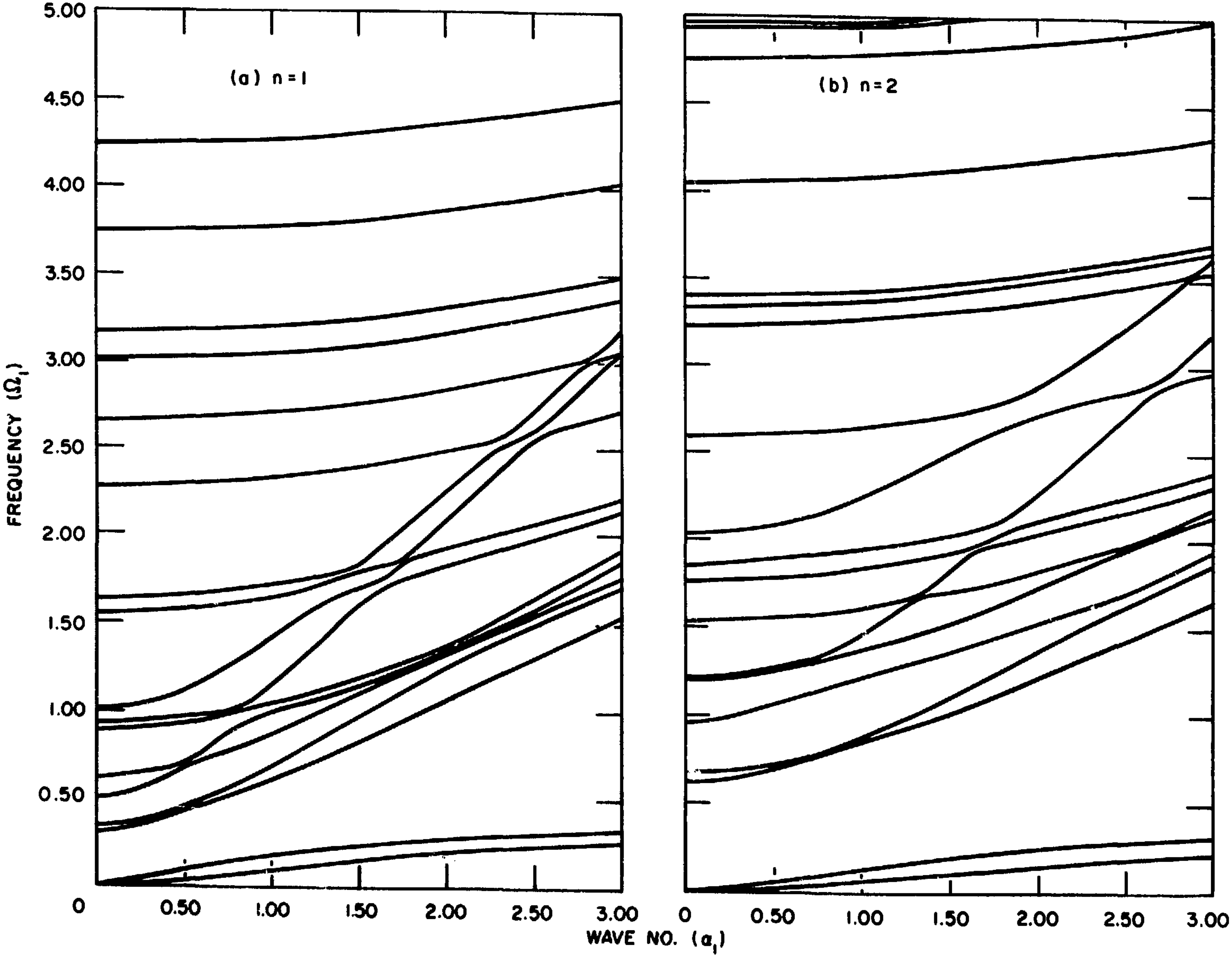

Fig. 4. Frequency Spectra of a Shell System Containing Compressible Fluid.

the fluid and shells is weak. The frequencies of structural modes are close to those of empty shells and the frequencies of the acoustic modes are close to those of a fluid cylinder and a fluid annulus enclosed by rigid walls. On the other hand, for large μ_i, the frequencies of structural modes decrease due to the increase of fluid loading and the acoustic mode frequencies approach those of a fluid cylinder and fluid annulus with pressure-release walls.

Consider the first two branches in Figs. 3 and 4, it is seen that the effect of fluid compressibility is to slightly lower the frequencies of the structural modes. In general, if the structural modes with low frequency are of interest, the fluid may be considered as incompressible.

IV. APPROXIMATE SOLUTIONS

It is of little trouble to obtain the roots of the exact frequency equation (17). However, it is still interesting to examine approximations to the low frequency for possible implications that may be deduced for the system.

Deleting the in-plane inertias of the shells, assuming that the fluids are incompressible, and using the Donell's shell equations (obtained by deleting all terms in Eq. (15) multiplied by δ_i^2, except the term $\frac{\delta_i^2}{12}(\alpha_i^2+n^2)^2$ in a_{16} and a_{26}), it can be shown that the radial displacements $\bar{w}_1$ and $\bar{w}_2$ are given by

$$\begin{bmatrix} \bar{\Omega}_1^2 - a\Omega_1^2 & b\Omega_1^2 \\ e\,\Omega_1^2 & \bar{\Omega}_2^2 - d\Omega_1^2 \end{bmatrix} \begin{Bmatrix} \bar{w}_1 \\ \bar{w}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{18}$$

where

$$\bar{\Omega}_1^2 = \frac{\delta_1^2}{12}(\alpha_1^2 + n^2)^2 + \frac{(1-\nu_1^2)\alpha_1^4}{(\alpha_1^2+n^2)^2}$$

$$\bar{\Omega}_2^2 = \frac{\delta_2^2}{12}(\alpha_2^2 + n^2)^2 + \frac{(1-\nu_2^2)\alpha_2^4}{(\alpha_2^2+n^2)^2}$$

$$\begin{aligned} a &= 1 + \mu_1 f_1 + \mu_1\left(\frac{\sigma_2}{\sigma_1}\right) f_1 \\ b &= \mu_1\left(\frac{\sigma_2}{\sigma_1}\right) f_5 \\ d &= (1 + \mu_2 f_3)k^2 \\ e &= \mu_2 f_4 k^2 \\ k &= \frac{R_2}{R_1}\left[\frac{\rho_2 E_1 (1-\nu_2^2)}{\rho_1 E_2 (1-\nu_1^2)}\right]^{1/2} \end{aligned} \tag{19}$$

Note that $\bar{\Omega}_1$ and $\bar{\Omega}_2$ are the dimensionless frequencies of the inner and outer shells in vacuo, respectively.

Consider two special cases: (a) the outer shell is rigid, and (b) the inner shell is rigid. Case a corresponds to a shell containing a fluid and submerged in a fluid annulus, while case b corresponds to a shell containing fluid and a rigid cylinder. The frequencies for these two cases are denoted by $\bar{\Omega}_{1i}$ and $\bar{\Omega}_{1o}$, respectively. From Eq. (18) it is seen that

$$\bar{\Omega}_{1i}^2 = \frac{\bar{\Omega}_1^2}{1 + \mu_1 f_1 + \mu_1 \left(\frac{\sigma_2}{\sigma_1}\right) f_2}$$

and (20)

$$\Omega_{1o}^2 = \frac{\bar{\Omega}_2^2}{(1+\mu_2 f_3)k^2}$$

It is obvious that both frequencies are reduced due to the fluid loading. It is useful to use the concept of added mass in this problem. In terms of dimensional quantities, from Eqs. (20) it can be shown that the frequency of the shell with fluid is equal to that of the empty shell whose density has been replaced by the shell density and the added mass. The added masses are

$$\rho_{added} = \frac{\sigma_1 R_1}{\rho_1 h_1} f_1 + \frac{\sigma_2 R_1}{\rho_1 h_1} f_2$$

for case a; and (21)

$$\rho_{added} = \frac{\sigma_2 R_2}{\rho_2 h_2} f_3$$

for case b. f_1, f_2, and f_3 are the added mass factors which are functions of n, α_i, and R_2/R_1. For $\alpha_i = 0$, Eqs. (16) give

$$f_1 = \frac{1}{n}$$
$$f_2 = f_3 = \frac{1}{n}\left[\frac{\left(\frac{R_2}{R_1}\right)^n + 1}{\left(\frac{R_2}{R_1}\right)^n - 1}\right] \tag{22}$$

Next, return to Eq. (18) concerning the coupling system. In this case, the frequency equation is

$$(ad + be)\Omega_1^4 - (a\bar{\Omega}_2^2 + d\bar{\Omega}_1^2)\Omega_1^2 + \bar{\Omega}_1^2\,\bar{\Omega}_2^2 = 0 \tag{23}$$

Equation (23) gives two frequencies; the smaller one Ω_{10} is associated with the out-of-phase motion and the larger one Ω_{1i} is associated with the in-phase motion of the two shells. Mathematically, it can readily be shown that if

$$\frac{be}{ad} < 1 \tag{24}$$

then

$$\Omega_{10} < \bar{\Omega}_{1i},\ \bar{\Omega}_{10}$$
$$\Omega_{1i} > \bar{\Omega}_{1i},\ \bar{\Omega}_{10} \tag{25}$$

Physically, this means that the freuqency of the out-of-phase mode is always less than those of the uncoupled system, while the frequency of the in-phase mode is always larger than those of the uncoupled system provided that

$$(1 + \mu_2 f_3)\left[1 + \mu_1 f_1 + \mu_1\left(\frac{\sigma_2}{\sigma_1}\right) f_2\right] > \mu_1\mu_2\left(\frac{\sigma_2}{\sigma_1}\right) f_4 f_5 \tag{26}$$

Equation (26) can be replaced by

$$f_2 f_3 > f_4 f_5 \tag{27}$$

Note that the coefficients f_2 and f_3 are proportional to the fluid pressure acting on the inner and outer shells due to the motion of the inner and outer shells respectively; while, the coefficients f_4 and f_5 are proportional to the fluid pressure acting on the outer and inner shells due to the motion of the inner and outer shells respectively. Equation (27) is found to be satisfied in all cases.

From Eqs. (20) and (25), it is clear that when either shell is rigid, the frequency of the other shell can be calculated in terms of the shell frequency in vacuo and the added mass factor. If both shells are elastic, the concept of added mass factor is no longer useful. However, when the frequencies of two shells are not close to each other, the shell with higher frequency may be considered as rigid in computing the lowest frequency of the coupled system. The frequency obtained in this way is the upper bound.

Finally, it should be mentioned that, in general, the error of Eq. (23) is small in the low frequency range. Moreover, the error decreases as $n\alpha_i$ increases and is negligible for large values of $n\alpha_i$. This is similar to that of a single shell containing a liquid [3].

V. APPLICATIONS

Turning now to a specific numerical example, consider the following dimensional values: $R_1 = 34.1875$ in.; $R_2 = 34.625$ in.; $h_1 = 0.25$ in.; $h_2 = 0.625$ in.; $E_1 = E_2 = 27.5 \times 10^6$ psi; $\nu_1 = \nu_2 = 0.27$; $\rho_1 = \rho_2 = 0.2885$ lb/in.^{3}; $\sigma_1 = \sigma_2 = 0.0333$ lb/in.^{3}; $\ell_1 = \ell_2 = 41$ in. The fluid is considered to be incompressible and the shell is assumed to be simply-supported at both ends. The frequencies of this finite shell system can be obtained using the frequency equation (17). The frequencies of the system depend on the axial wave number and circumferential wave, the lowest frequency is associated with the lowest axial wave number; i.e., the shell length is equal to the half wave-length. The frequencies of the out-of-phase and in-phase modes for this case have been computed and presented in Fig. 5. For comparison, Fig. 5 also shows the frequencies for four related cases: (1) the inner shell in vacuo; (2) the outer shell in vacuo; (3) the shell system with rigid outer shell; and (4) the shell system with rigid inner shell.

These examples illustrate the dynamic characteristics of a coupled shell-fluid system as discussed previously. Another character exhibited in this figure is that the circumferential wave number associated with the lowest frequency for a coupled shell-fluid system, in general, is different from that of an empty shell. As shown in the figure, this circumferential wave number of the inner shell changes from 7 to 6, while that of the outer shell does not change. In the case of the double shell system, the lowest frequency of the out-of-phase mode is associated with $n = 6$, while the in-phase mode with $n = 5$. This behavior is attributed to the fact that the distribution of stretching energy and bending energy of the shell-fluid system is different from that of an empty shell.

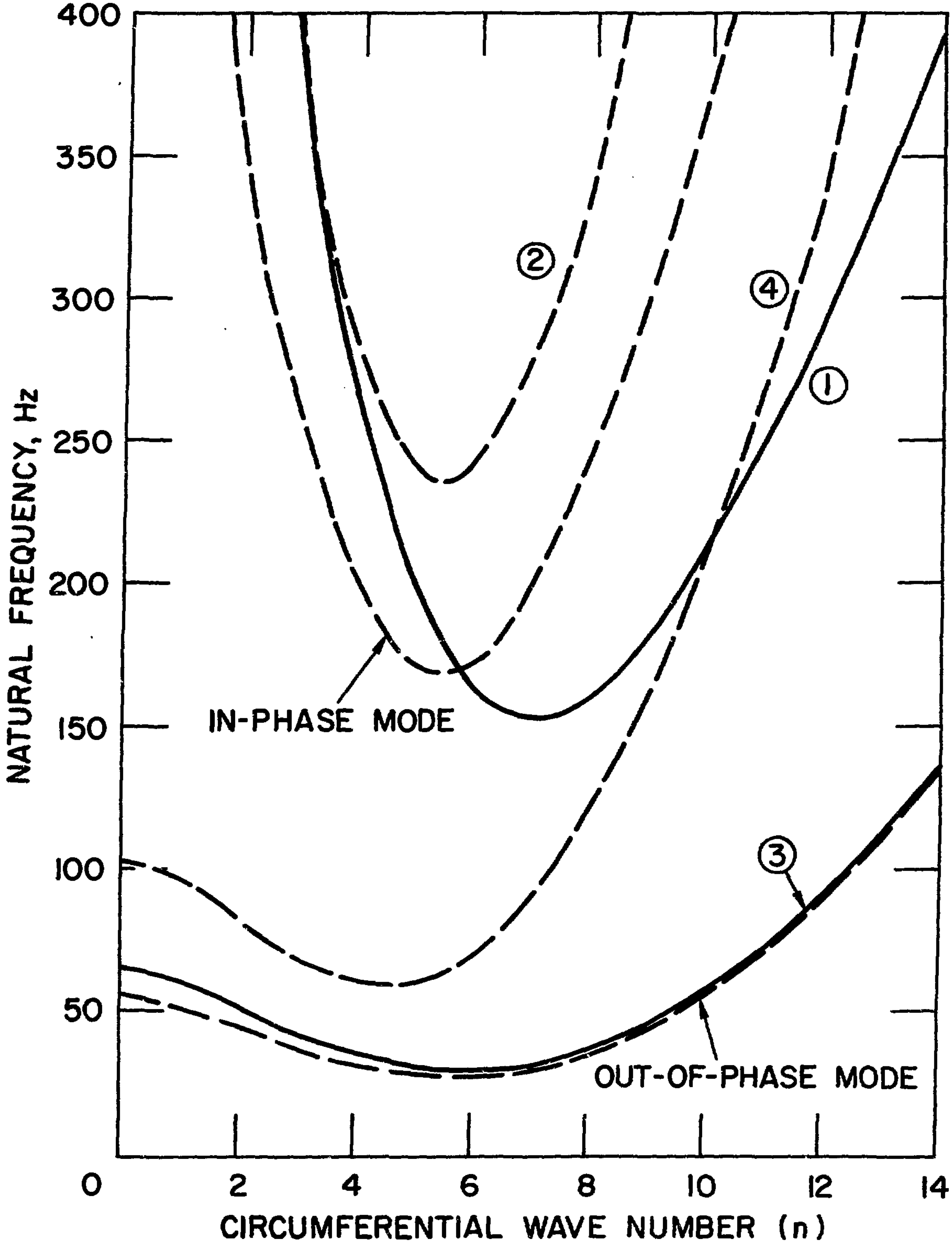

Fig. 5. Natural frequencies of out-of-phase and in-phase modes of a coupled shell system and related cases: (1) inner shell in vacuo; (2) outer shell in vacuo; (3) the shell with rigid outer shell; and (4) the shell system with rigid inner shell.

VI. CONCLUSIONS AND DISCUSSIONS

A general method for calculating the frequencies of a coupled shell/fluid system is presented in the paper. Following are some important conclusions: (1) There exist structural modes and acoustic modes. If the structural motions are of primary interest, the fluid may be considered as incompressible. (2) There exist out-of-phase modes and in-phase modes. The lowest frequency of the system is always associated with one of the out-of-phase modes. (3) The lowest frequency of the shell system with fluid is significantly lower than those of the individual shells. (4) The manner of accounting for the effect of the fluid coupling via the added mass concept is described explicitly. (5) The distribution of stretching energy and bending energy of the shells within the coupled system is different from those of the corresponding empty shells. With the suggested method, the frequency characteristics of thermal shield can be analyzed and design parameters can be explicitly related to frequency.

The results presented in this report are useful in design and evaluation of systems containing circular cylindrical shells, such as thermal liners. In practice, Eq. (23) can be used to find the frequency of a coupled shell system containing fluid. Since a closed form solution is given, parametric study can be made easily.

The coefficient f_1, f_2, f_3, f_4, and f_5 are added mass coefficients, which depend on circumferential wave number n, axial wave number α_i, and radius ratio R_2/R_1. These coefficients will be used in the development of sets of design curves to be included in a future design guide.

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