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Exponential Families of Stochastic Processes

Springer

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Preface

Exponential families of stochastic processes are parametric stochastic process models for which the likelihood function exists at all finite times and has an exponential representation where the dimension of the canonical statistic is finite and independent of time. This definition not only covers many practically important stochastic process models, it also gives rise to a rather rich theory. This book aims at showing both aspects of exponential families of stochastic processes.

Exponential families of stochastic processes are tractable from an analytical as well as a probabilistic point of view. Therefore, and because the theory covers many important models, they form a good starting point for an investigation of the statistics of stochastic processes and cast interesting light on basic inference problems for stochastic processes.

Exponential models play a central role in classical statistical theory for independent observations, where it has often turned out to be informative and advantageous to view statistical problems from the general perspective of exponential families rather than studying individually specific exponential families of probability distributions. The same is true of stochastic process models. Thus several published results on the statistics of particular process models can be presented in a unified way within the framework of exponential families of stochastic processes.

The exponential form of the likelihood function implies several probabilistic as well as statistical properties. A considerable portion of the book is focused on clarifying such structure of exponential models. Other main themes are asymptotic likelihood theory and sequential maximum likelihood estimation. These areas of statistical inference for stochastic processes are basically different from the similar problems for independent observations. In particular, in the asymptotic likelihood theory, the dependence structure of a process class can imply a wealth of interesting new situations.

In recent years, stochastic calculus has been used increasingly to study inference problems for stochastic processes, and there is scope for using this powerful tool to a much larger extent. A major obstacle to this development is that stochastic calculus is not widely known among statisticians. It is hoped that this book will assist graduate students as well as researchers in statistics not only in getting into the problems of inference for stochastic processes by studying the most tractable type of models, but also if necessary in learning to solve the problems by the tools of stochastic calculus. To attain this goal, the first chapters use only classical stochastic process methods, while tools from stochastic calculus are used at the end. The necessary tools from stochastic calculus are reviewed in an appendix. Most chapters include exercises to support the learning process. We also hope that students and researchers in probability can use the book to get acquainted with the problems of statistical inference.

The authors acknowledge with gratitude the financial support to their collaboration on this book from the EU programme Human Capital and Mobility and from Sonderforschungsbereich 373 at the Humboldt University in Berlin, which among other things made it possible for Michael Sørensen to stay for a longer period in Berlin. We are particularly grateful for the support from the Volkswagen-Stiftung through the programme "Research in Pairs" at the Mathematical Research Institute Oberwolfach, without which it would have taken a lot longer to finish this book. In fact, a large portion of this book was written at Oberwolfach. At an early stage in the writing of the book, an invitation to Michael Sørensen to give a series of lectures on the subject of the book at IMPA in Rio de Janeiro resulted in a pleasant stay that gave much inspiraton to go on with the project.

We would also like to thank the many colleagues who have contributed to our book project by writing papers or otherwise collaborating with us, by sending us preprints, by discussing the topic with us, or by giving helpful comments on our papers and on the book manuscript in its various stages. Whatever errors there remain are, of course, our responsibility. We hope the reader will contact us with any questions, comments, or criticisms she or he might have.

Four secretaries, Ms. O. Wethelund and Ms. H. Damgaard at the University of Aarhus and Ms. S. Bergmann and Ms. A. Fiebig at the Humboldt University in Berlin, typed the first version of most of the book into the computer and assisted us in organizing the activities related to the book project. We are grateful for their invaluable work of outstanding quality.

Finally, we dedicate this book to Ingeborg and Ulla for their care, support, and tolerance.

Uwe Küchler and Michael Sørensen

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