Where form and substance meet: Using the narrative approach of *re-storying* to generate research findings and community rapprochement in (university) mathematics education

Elena Nardi

School of Education and Lifelong Learning, University of East Anglia, Norwich NR47TJ, United Kingdom

e.nardi@uea.ac.uk, +441603592631

Abstract

Story-telling is an engaging way through which lived experience can be shared and reflected upon, and a tool through which difference, diversity – and even conflict – can be acknowledged and elaborated upon. Narrative approaches to research bring the richness and vibrancy of story-telling into how data is collected and interpretations of it shared. In this paper I demonstrate the potency of the narrative approach of *re-storying* for a certain type of university mathematics education research (*non-deficit*, *non-prescriptive*, *context-specific, example-centred* and *mathematically-focused*) conducted at the interface of two communities: mathematics education and mathematics. I do so through reference to *Amongst Mathematicians* (Nardi, 2008), a study carried out in collaboration with 20 university mathematicians from six UK mathematics departments. The study deployed *re-storying* to present data and analyses in the form of a *dialogue* between two fictional, yet entirely data-grounded, characters – M, mathematician, and RME, researcher in mathematics education. In the dialogues, the typically conflicting epistemologies – and mutual perceptions of such epistemologies – of the two communities come to the fore as do the feasibility-of, benefits-from, obstacles-in and conditions-for collaboration between these communities. First, I outline the use of narrative approaches in mathematics education research. Then, I introduce the study and its use of *re-storying*, illustrating this with an example: the construction of a *dialogue* from interview data in which the participating mathematicians discuss the potentialities and pitfalls of visualization in university mathematics teaching. I conclude by outlining *re-storying* as a vehicle for community rapprochement achieved through generating and sharing research findings – the *substance* of research – in *forms* that reflect the fundamental principles and aims that underpin this research. My conclusions resonate

with sociocultural constructs that view mathematics teacher education as contemporary *praxis* and the aforementioned inter-community discussion as taking place within a *third space.*

Keywords: narrative inquiry; re-storying; dialogic format; mathematicians; university mathematics education

The relationship between mathematicians and mathematics educators has been the focus of debate since at least the 1990s. Anna Sfard's (1998) discussion with Shimshon A. Amitsur – presented in the form of a dialogue – is one of the first. Writings by authors from a variety of geographical and institutional contexts such as Michèle Artigue (1998), Anthony Ralston (2004) and Gerry Goldin (2003) have portrayed this relationship as at best fragile. *Amongst Mathematicians* (Nardi, 2008) – the dialogic format of which (see Figure 1 for a sample page) this paper uses as an illustration – acknowledges this fragility and explores this relationship in the form of fictional yet data-grounded dialogues between a mathematician and a mathematics educator. The dialogues are composed out of lengthy interviews with 20 mathematicians based in the UK and deploy the narrative approach of *re-storying* (Ollerenshaw & Creswell, 2002). In this paper I exemplify and justify the use of this approach in a (university) mathematics education research context and I propose this use as a vehicle for a much needed inter-community partnership. First, I outline the use of narrative approaches in mathematics education research. I then introduce the context, participants and data of the study – and elaborate and exemplify how I deployed *re-storying* for the analysis of the data and the composition of the dialogues. I conclude with a discussion of how generating and sharing research findings – the *substance* of research – in *forms* that reflect the fundamental principles and aims of this research serves the purpose of inter-community partnership.

1. NARRATIVE APPROACHES IN MATHEMATICS EDUCATION RESEARCH

The roots and growth of narrative inquiry. Qualitative data analysis aims to produce generalisations embedded in the contextual richness of individual experience (Denzin & Lincoln, 2011). Coding and categorising techniques (Charmaz, 2005), a significant part of the canon of qualitative data analysis, often result in texts sorted into units of like meaning, with evident benefits including facilitated access to interpretation, inference and generalization. Narrative approaches (e.g. Connelly & Clandinin, 1990; Clandinin & Connelly, 2000) have the potential to take these benefits even further by generating holistic accounts with distinct contextual richness. Many authors (e.g. Ricoeur, 1984/1985/1988) acknowledge the narrative ways in which we understand our self, the others and the world we live in. Qualitative research, with its growing appreciation of narrative approaches as a research tool, has been increasingly mirroring this acknowledgement (Webster & Mertova, 2007; Lieblich, Tuval-Mashiach & Zilber, 1998). The roots of narrative enquiry can be traced within and across several disciplines (Clandinin, Pushor, & Murray Orr, 2007), including cultural studies (Andrews, 2006), folklore studies (Barrett & Stauffer, 2009), anthropology (Bateson, 1994), sociology (Carr, 1986) and psychotherapy **(**Schafer, 1981**)**. It is reasonable to claim that, even though still emerging as a field (Chase, 2011), narrative research now sits comfortably alongside phenomenology, grounded theory, case study and ethnography as a core paradigm of qualitative inquiry (Clandinin, 2008).

Narrative inquiry in education and in mathematics education research. Narrative inquiry has been gaining ground in educational research – with a focus being mainly on the practices of teachers and teacher educators as well as on the interface between the lives of children and teachers – often through the extensive and influential work of D. Jean Clandinin and F. Michael Connelly (e.g. Connelly & Clandinin, 2005). In mathematics education, many researchers have deployed a variety of narrative approaches to explore: children's mathematical growth (Burton, 2002); mathematics teachers' trajectories as they enter the profession (Frost, 2010); young people's evolving mathematical identities (Darragh, 2013), especially in relation to gender (Solomon, 2012) and to representations of mathematics in popular culture (Moreau, Mendick & Epstein, 2010); teachers' and learners' ways of relating to new technologies (Healy & Sinclair, 2007); and, educational evaluations across curricular, social and cultural contexts (Cantú, 2012).

A perspective on narrative that resonates with the research discussed in this paper is in the study by Healy & Sinclair (2007), particularly their take on Bruner's distinction between *narrative* and *paradigmatic* ('logical/classificatory one that has typically been associated with mathematics', p. 5) *modes* of how humans experience and account for the world. Of specific interest to this paper is the narrative approach of *re-storying* as defined by Ollerenshaw and Creswell (2002) and closely associated with the characteristics of narrative introduced by Bruner (1991) – drawn upon in the aforementioned study by Healy & Sinclair (2007) and elaborated upon in Nardi (2008, pp. 20-21). I note that, while these authors focus on the stories that mathematicians, and learners, tell as they engage with *mathematics*, the use of narrative approaches discussed here focuses on the stories told by mathematicians as they engage with conversation on *the teaching and learning of mathematics.*

The narrative approach of re-storying. Re-storying is the process of constructing a 'story from the original data' (Ollerenshaw and Creswell, 2002, p.330) based on 'narrative elements such as the problem, characters, setting, actions, and resolution' (p.332). Analysis often involves familiar qualitative approaches such as theme, pattern or causal-link identification. The account of the researcher's own gaining of insight into the data is often also interwoven in the construction process and is visible in the newly-constructed story. In a nutshell, the process of *re-storying* involves: becoming familiar with raw data (such as interview transcripts, participant diaries etc.); identifying the elements of a new story to be told out of the stories of the participants; and, then, composing the new story. A distinctive element of the new story is that if it "merely recounts a sequence of events, without evaluating or interpreting it, then it cannot be counted as a story" (Healy & Sinclair, 2007, p.19). As Clandinin & Connelly have often written (e.g. 2000), the processes through which the new stories are generated can be complex – as is the task of presenting a *researcher's account* of these processes that is transparent and open to scrutiny and replication. I see this paper as a modest contribution towards a collection of such researcher accounts.

One challenge that a presentation on the *re-storying* approach has to tackle is the view that data analysis is by definition a form of re-storying – as in the stories that participants and researchers co-construct in the course of data collection (e.g. interviews). The particular take on the *re-storying* approach presented here assimilates the multiplicity of voices (researchers' and research participants', as well as amongst the participants themselves) without suppressing or eliminating this multiplicity. Furthermore the transparency of the process, as showcased in the example presented in Section 4, renders this process accountable and replicable.

Beyond this methodological rationale for describing how I have used *re-storying* in Nardi (2008) there also lies an epistemological and pragmatic purpose. I see the stories that can be told in this manner as a potent communicative tool which can be deployed by two communities – mathematics and mathematics education research – which often find communication challenging (Artigue, 1998). The main claim I make here is that the stories that this approach generates –

directly relevant, mathematically-focussed, jargon-free, yet underpinned by an awareness of findings from research into the teaching the learning of mathematics at university level – can help fulfil the pedagogical potential which lies within this often challenging partnership. I return to this claim in my conclusion.

In what follows, I elaborate my adaptation of *re-storying* in the analyses in Nardi (2008), presented in a less common, but not unprecedented, format: a *dialogue* between two fictional, yet entirely data-grounded, characters (M, mathematician, and RME, researcher in mathematics education), as illustrated in Figure 1.

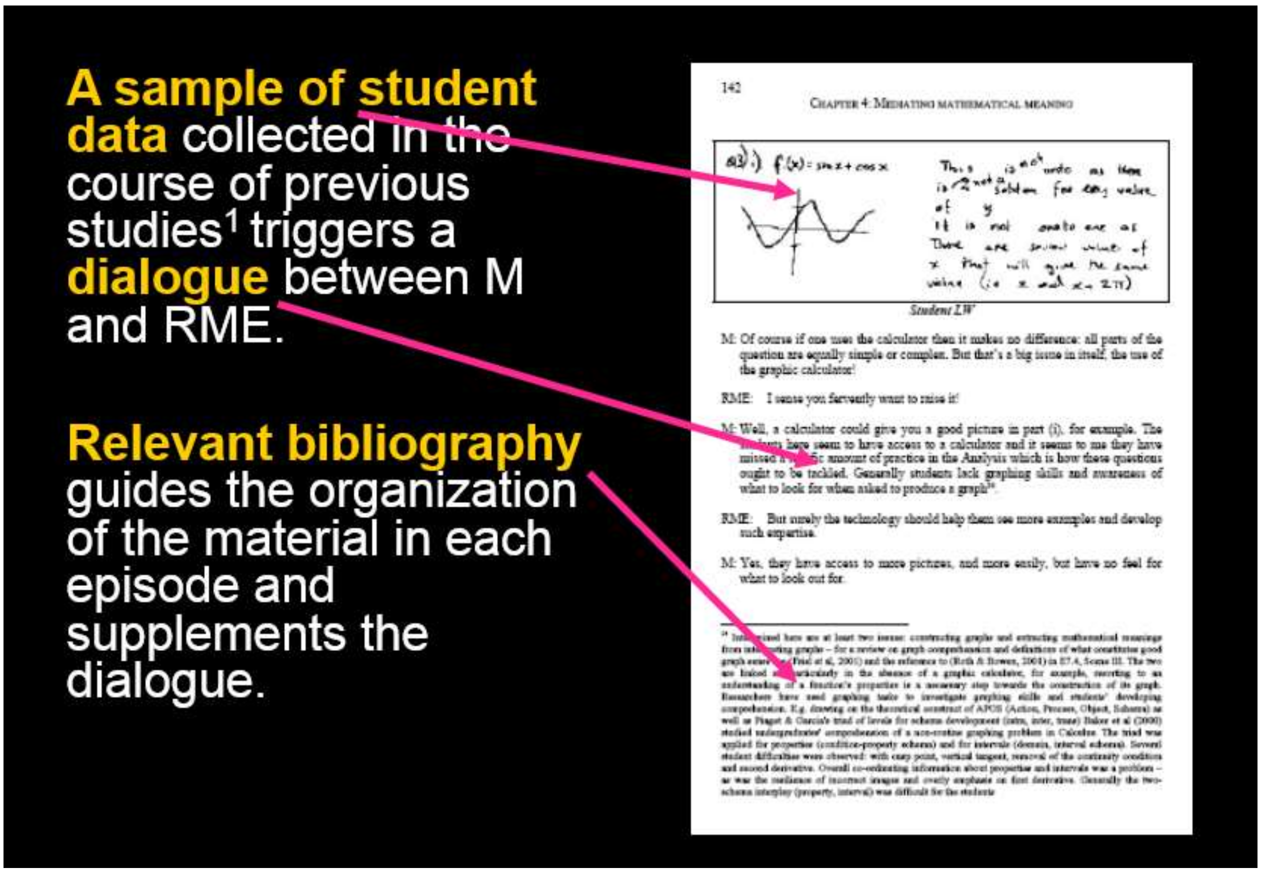


Figure 1. *Amongst Mathematicians,* sample page.

2. A *RE-STORYING* STUDY OF UNIVERSITY MATHEMATICS TEACHING: DATA AND PARTICIPANTS

The dialogues between M and RME in *Amongst Mathematicians* are fictional, yet data-grounded, constructed from the raw transcripts of the interviews with university mathematicians and then thematically arranged in *Episodes*. The interviews were eleven audio recorded half-day focus group interviews with twenty pure and applied mathematicians from six UK mathematics departments, and conducted by myself and Paola Iannone, co-investigator in this study. All were male, white and European, with one exception. Many had significant international experience and their age ranged from early thirties to late fifties. Their teaching experience varied from a few years to a few decades. Discussion was triggered by data samples consisting of students' written work, interview transcripts and observation protocols collected during (overall typical in the UK) Year 1 introductory courses in Analysis / Calculus, Linear Algebra and Group Theory.

Each *Episode* sets out from a discussion of a data sample, distributed to participants at least a week in advance, which typically operates as a trigger for addressing an issue on the learning and teaching of mathematics at the undergraduate level. Samples included students' written work, interview transcripts, or observation protocols collected in the course of prior studies (Nardi, 2008; p. 12-13), which had emerged as typical in the course of data analysis. Most concerned students' learning experiences in introductory Year 1 or 2 undergraduate mathematics courses (mostly standard parts of introductory courses in Analysis or Calculus, Linear Algebra and Group Theory), and comprised approximately 16-page booklets containing about half a dozen examples. Each example consisted of an item from a problem sheet, its suggested solution by the lecturer leading the module and one or more student responses, largely typifying issues selected as worthy of further consideration. Some of these issues were listed succinctly after the examples. The participating mathematicians generally recognised the material discussed in the samples as typical of students' early experiences of university mathematics (in the UK). An excerpt from one of the data samples distributed to the participants prior to the interviews is in the Appendix.

Most participants arrived at the interviews – which explicitly aimed at eliciting and exploring their pedagogical perspectives – with comments and questions scribbled in the margins and eager for a close examination of the data samples. The study was conducted in full awareness of the potential "discrepancy between theoretically and out-of-context expressed teacher beliefs about mathematics and pedagogy (e.g. in interview-based studies) and actual practice" (Biza, Nardi & Zachariades, 2007, p. 301), and at least three of the five characteristics in the study's research design (*non-deficit, non-prescriptive, context-specific, example-centred* and *mathematically-focused*) were deliberately put in place in order to curtail this discrepancy.

However, my collaborators and I remained fully aware that the interviews elicited participants' *stated* beliefs and *intended* practice.

Data analysis – described in Sections 3 and 4 of this paper – resulted in thematically linked sequences of *Episodes*, the majority starting with the discussion of an excerpt from the data sample booklets. In the course of the interviews (and, as a consequence, in the resulting *Episodes*) the researchers (the character of RME) presented the participating mathematicians (the character of M) with more student responses. Participants suggested additional examples that were also incorporated in the *Episodes*. Throughout, they seemed fully aware of the overall aim of the study and their keen participation indicated that they were particularly willing to debate (often controversial aspects of) mathematical pedagogy at university level, both with the researchers and amongst themselves.

In the following, I outline the theoretical origins and rationale for my adaptation of the narrative approach of *re-storying* in the data analysis. Then, I describe how the raw data (interview transcripts and student data samples) were turned into the dialogues between the characters of M and RME that form the bulk of the text in Chapters 3-8 in Nardi (2008). In so doing, I aim to argue the central point of this paper: that the *form* (the dialogue between M and RME) in which the *substance* of the research (design in terms of these five characteristics: *non-deficit*, *non-prescriptive*, *context-specific*, *example-centred* and *mathematically-focused*; collecting and analyzing the data) is presented contributes to the rapprochement between the communities of mathematics and mathematics education.

3. *RE-STORYING* AND THE DIALOGIC FORMAT IN A STUDY OF MATHEMATICIANS' PEDAGOGICAL AND EPISTEMOLOGICAL PERSPECTIVES: TWO SENSES OF 'STORY'

The primary use of the term *story* in the work I present in this paper is the technical / methodological one that I describe in the preceding sections: the stories (dialogues) that constitute the bulk of the text in *Amongst Mathematicians* are the output of my endeavour to make sense of how the participants articulate their experiences of, and views on, the teaching and learning of mathematics. In addition to this use of the term, a further notion of *story* emerged in the course of the study which aligns well with discursive approaches to the study of mathematics teaching and learning (generally: Sfard, 2008; specifically to university mathematics: Nardi, Ryve, Stadler & Viirman, 2014): the mathematicians have their own ‘stories’, their own ways of articulating how they make sense of their students’ learning and their own pedagogical practices, and how they relate to the *colloquial* and *literate* discourses (Sfard, 2008, p. 299) of mathematics education as a discipline. The term *discourses* here covers both *ways of speaking about* and the *practices of* mathematics education. As an example, in Nardi & Iannone (2003), we wrote about the way that mathematicians used words such as ‘landscapes’ (and other more or less synonymous words) to describe their students’ emerging mathematical perceptions. We were struck by how closely their use evoked that of Tall and Vinner’s (1981) use of the term *concept image* and it seemed to us that this use often characterised the ‘stories’ that these interviewees were telling about their students’ learning. Analysis of the data for the narratives presented in Nardi (2008) revealed other such ‘stories’: ‘mathematics as a language to master’ (e.g., when interpreting students’ written or verbal communication), ‘gradual and negotiated induction into the practices of university mathematics through interaction between experts and newcomers’ (e.g., when stating preferred teaching practices); and ‘us and them/you’ (e.g., when expressing caution, even apprehension, towards the mathematics education community).

The two senses of *story* outlined above – the technical/methodological one that is aligned with my use of the *re-storying* approach and the interviewee-originating one that is aligned with their ways of seeing and speaking about the teaching and learning of mathematics – are distinct but, also, inevitably and deliberately interrelated. How I chose the gist of the stories in *Amongst Mathematicians* was partly driven by the ‘stories’ discerned in the interviewees’ utterances. I elaborate on this process in Section 4.

In tandem with the influences briefly outlined so far, my choice of the dialogic format is based on its natural affinity with that of which M and RME speak in *Amongst Mathematicians*. Mostly associated as a format for philosophical texts, the dialogic format was imported to mathematics education most famously by Imre Lakatos’ *Proofs and Refutations* (1976), a fictional dialogue set in a mathematics classroom which features students’ attempts to prove the formula for Euler’s characteristic. Through their successive attempts, the students re-live the trials and tribulations of the mathematicians who had previously attempted this proof – largely through the successive construction of key counterexamples. The way Lakatos assimilates the multiplicity of often conflicting perspectives without suppressing or de-valuing it but, instead, fleshing it out – was a strong influence on the construction of the dialogues in Nardi (2008).

Another influence originates in a certain school of contemporary theatre, literature and film exemplified by Michael Frayn's re-imaginings of key scientific, political or artistic encounters in plays such as *Copenhagen* (1998), and Tom Stoppard's complex, multi-layered discursive shifts in plays such as *Arcadia* (1993). Within education, a fundamental influence (Nardi, 2008, p.20) was Jerome Bruner's (1991) ten characteristics of narrative – *diachronicity* (events occur over a period of time); *particularity*; *intentional state entailment* (characters have beliefs, desires, theories, values etc.); *hermeneutic composability* (narratives can be interpreted as playing constitutive role in a 'story'); *canonicity and breach* (stories can be about 'breaches' of normal, canonical states); *referentiality* (a story references reality although it may not offer verisimilitude); *genericness* (flipside to particularity, paradigmaticity); *normativeness* (linked to 'canonicity and breach', about how one *ought to* act); *context sensitivity and negotiability* (relating to hermeneutic composability and defining the contextual boundaries within which the narrative works); and *narrative accrual* (stories are cumulative).

In the following section I describe the process through which the dialogues between M and RME came to be, as an assimilation of the multiplicity of voices while foregrounding the participants' perspectives. The focus of the dialogue is deliberately on M, the 'role' of RME being kept to a minimum, in symmetry with how the original interviews were conducted. This (quantitatively) minimal presence of RME in the dialogues can be a little misleading: the influence of researcher perspectives on the choice of themes of the *Episodes*, and in the clusters of *Episodes* that became C*hapters* in *Amongst Mathematicians* has been very substantial. In fact it is this fusion of researcher and participant perspectives that my argument for the *re-storying* approach highlights.

4. FROM INTERVIEW TRANSCRIPTS TO DIALOGUE: APPLYING A *RE-STORYING* APPROACH IN A STUDY OF MATHEMATICS TEACHING AND LEARNING

The eleven focus group interviews produced an average transcript length of 35,000 words, in which the order of discussion usually followed the structure of the data samples (the sequences of mathematical problems / solutions / typical student responses / issues to consider) that had been distributed. Across the transcripts, 25 data samples were discussed (each at least twice). I created 25 folders, one for each data sample, which contained the full transcript, descriptive summaries of the parts in which the data sample had been discussed and scanned images of relevant materials (participants' writing during the interviews, other student data discussed during the interview etc.). The materials within each folder formed the basis for a 'field text' (Clandinin & Connelly, 2000, p. 92) – *Narrative* thereafter – which included: the mathematical problem and its recommended solution; the student responses that had been used as triggers for discussion; a list of issues that the interviewees had been asked to consider; and a dialogue between two characters, M and RME, each consolidating the contributions in the interviews by the participating mathematicians (for M) and the researchers conducting the interviews (for RME).

The dialogue consists of M and RME's utterances, where M's utterances are a consolidation of verbatim quotations from the twenty participating mathematicians and RME's utterances are a consolidation of the minimally leading interventions of the researchers during the focused group interviews. The links between the dialogue and relevant literature are in the form of footnotes. While I would not want to suggest that one unified perspective on M, RME and the literature is possible – or even desirable – the aim of this approach is to contribute to the substantive conversation regarding the teaching and learning of mathematics at university level by bringing to the fore M's views on and experiences of these issues, and to represent the complexity and sensitivity of their pedagogical perspectives. The 25 *Narratives* evolved into the 24 *Episodes* presented in *Amongst Mathematicians*. Sometimes also broken in *Scenes*, *Episodes* start with a mathematical problem and (usually) two student responses. A dialogue between M and RME on issues exemplified by the student responses ensues. Other examples of relevant student work are interspersed in the dialogue. I now outline the process of converting the *Narratives* into *Episodes.*

The *Narratives* contained the first attempts at converting the material from each sample into a dialogue between M and RME, and led to an increasing understanding of the themes and issues the dialogues were revolving around. The aim was to present the dialogues in the *Narratives* – which, in the natural course of conversation in the interviews, ebbed and flowed across many different issues – so that the strength of the material (authenticity, richness and naturalistic flow) could be maintained while offering the reader a sense of focus, structure and direction(s) towards which the conversation is heading. The final process involved sharpening the focus of the *Narrative* until it is about a tangible focal point, and then rewriting the *Narrative* in accordance with the following five steps: (1) introduce the focal point with reference to previous studies and justification of its significance; (2) zoom the dialogue in on those parts where M makes a substantial contribution relating to said focal point; (3) abbreviate the rest but do not eliminate (to secure continuity and flow of the *Narrative*) signaling to the reader that such abbreviation is taking place; (4) strengthen the visibility of links to related literature with further references in the footnotes; and (5) conclude with a brief reflective comment on the preceding dialogue.

Several distillations and rearrangements of the *Narratives* followed and led to the thematic breakdown of the data and findings presented in Chapters 3-8 in Nardi (2008), focusing on: mathematical reasoning, conveying mathematical meaning, functions, limits, pedagogy, and the M-RME relationship. One noteworthy observation that emerged in the course of this distillation process is that a substantial part of the material in the *Narratives* did not fit neatly within the thematic clusters of the grander narrative of the analysis. Some of this material became the *Special Episodes* and *Out-Takes* in Chapters 3-8. Throughout this process, the *Narratives* were maintained as solidly immersed in the specificity of the discussion in the interviews and steered towards citing the relevant literature and theorising from this side-by-side citation. Whether a narrative achieved this aim of simultaneous specificity/data-groundedness and generalisation became a determinant of what stays and what goes.

Within every step of the data consolidation process I have outlined in this section, there are perils as well as benefits. So, for example, alongside the obvious benefit of streamlining the data to the extent of making it – and its analysis – more communicable, there is the potential loss of nuance in the ways difference / conflict is transformed from the raw data and reflected into the re-storied narrative. I address the benefits as well as some of the perils in Section 5 through the discussion of an example of re-storying interview data.

5. AN EXAMPLE OF RE-STORIED INTERVIEW DATA

In this section I illustrate the application of the *re-storying* approach described in Section 4, through an example of how a small number of interview excerpts were *re-storied* into one exchange of utterances between the characters of M and RME. The exchange can be found in Nardi (2008, p. 143):

[RME invites views on students' use of information compressed in a function graph]

M: I encourage them to draw graphs, see what the answer is and then prove it afterwards. The graph is fantastic to get the answer but there is actually not enough in their writing, once they have done that. It may be a bit of a surprise that I do, even though in the suggested solutions in this particular question you don't see much resorting to graphs. It probably does say quite a lot that the lecturer thinks in terms of domain etc. and not overtly about graphs. I would be drawn towards a low-tech approach, roughly draw them and insert them on the side but I wouldn't find them necessary for answering this question. I would like the students though to carry the graphs of all these functions in their heads straightaway and have them immediately available. But then again there are less and more visual people and the more visual may think that a question like this can only be done by producing the graph, using it and then proving the claim formally. Graphs are good ways to communicate mathematical thought and I do not wish to underplay that at all.

RME: Are you worried when the students rely too much on the graph in order to demonstrate their claims?

[The Episode continues with M turning to the response by Student WD in the data sample in order to discuss this issue.]

This exchange is from *Episode 4.3* entitled *Visualisation and the role of diagrams* (Nardi, 2008, p. 139-150). This *Episode* originates in one of the 25 *Narratives* (see Section 4) which bring data together under broad thematic clusters; in the case of this particular *Narrative*, this was "the use of graphs and graphic calculators in mathematical reasoning, [teaching: is absolute rigour pedagogically viable?]".

The exchange comes after the interviewees were asked to consider the data sample in the Appendix, which was discussed in two of the eleven interviews. Prior to commenting separately on each of the responses of Students WD and LW, the interviewees choose to discuss more broadly the usefulness of graphs in discerning properties of functions such as *injectivity* and *surjectivity*. Here is the relevant transcript excerpt from one of the two interviews. Utterances are numbered according to the following rules: RME_2 and RME_1 are the two researchers conducting the interviews; interviewees have been anonymized as M_S, M_M, M_J, M_T and M_P; and, the last number in the subscript denotes the turn number, i.e. RME_{21} is the first turn of RME_2, M_{M2} is the second turn of M_M etc. I note that – as exporting and importing utterances across *Episodes* risks losing grip of the context in which an utterance was made in the first place – the exact numbering of utterances allows what was said, by whom and where, to be traced quickly and efficiently:

RME_{21}: […] what we wanted to look at in this context was the use of graphs. How good….how productive or unproductive is the use of graphs for a question of this type.

M_S: I would encourage them to draw graphs and see what the answer is and then prove it afterwards. The graph is fantastic to get the answer but there is actually not enough, once they have done that.

M_{M1}: I am surprised that the lecturer is suggesting that some students have any pictures in it at all.

M_J: Well, difficult to do.

RME_{11}: Yes…
M_{M2}: Right.
(pause)
RME_{22}: Well, but you would be surprised to see… because…
RME_{12}: It is notes on solutions, it is not …the complete….
M_{T1}: Yes…
M_{P1}: It does say quite a lot, it is not graphs, it is domain and …
M_{T2}: Unless you do them by hand. I just would insert the gaps and fill them in, in black pen for such a question… pretty low tech way. But this again shows that as soon as there is something to do they think, oh yes, I think … of course it is not so serious but the question is partly where these graphs are located. I mean…Sometimes you would question at level to have graphs but in their heads maybe, it is not necessary. I mean, it is fine to draw them but it is not necessary. And… you know…that too to me is something that they should aspire to have… graph of sin*x* … it is fine to draw it but also you should carry it in your head, always being able to visualize every single graph…. Is the obvious thing, I guess.
M_{P2}: I always saw mathematics and the different ways that people learn, some people are visual, other people are not visual. So if that provides that framework to students to begin with and then let them follow their own… and they may show the formal answer at the end… Well, I cannot do this sort of questions without sketching a graph.
M_{M3}: Quite, yes… But as we were talking earlier that communication means that students doing a sketch does encourage me a lot. Now we give them at least some marks for this… and I would rather be in the situation where that is part of the background noise and the student is at least thinking a bit about it.
(pause)
RME_{13}: So probably the difference to be doing the opposite of someone saying … that the students rely on the graph more than we would like probably …
[M_M , and then all, turns to Student WD's response to discuss this further.]

In the summary account of the data that comprised this particular *Narrative*, the interview excerpt above appears as follows:

> RME_2 introduces [this example from the Data Sample] and invites the group's comments on the students' use of graphs. M_S says he encourages the students to produce graphs in order to get the answer but then prove the statement formally. M_M is surprised at the lecturer's claim that the students use graphs at all and M_P observes that the notes on solutions include no graphs. M_T says that in a question like this, it is good to have a graph but not necessary, even though overall you need to be able to visualize every function. M_P disagrees: there may be less and more visual people but for him a question like this can only be done by producing the graph, using it and then proving the claim formally. M_M agrees that graphs are good ways to communicate mathematical thought. RME_1 suggests that it may be worrying when the students rely too much on the graph for demonstrating their claims. M_M, and then all, turns to Student WD's response.

The exchange that we eventually see on p. 143 in Nardi (2008) is *re-storied* from this summary account, and from analogous supportive evidence from the other interview where this data sample was discussed. Here are some examples of this supportive evidence (originating in utterances by participants M_R and M_I; only summarized here due to limitations of space):

M_R: [there is the] possibility of drawing (and inferring from) a wrong picture
M_{I1}: [some student pictures] are almost perfect but offer no construction evidence

M_{I2}: pictures are a good start.
M_{I3}: [we must] write out the answers explicitly and not in the condensed version of the lecturer's response.

The *re-storying* of the evidence into the utterance of M took place as follows. The utterance consists of the following component clauses, C1-C9:

C1: I encourage them to draw graphs, see what the answer is and then prove it afterwards.
C2: The graph is fantastic to get the answer but there is actually not enough in their writing, once they have done that.
C3: It may be a bit of a surprise that I do [encourage them to draw graphs] even though in the suggested solutions in this particular question you don't see much resorting to graphs.
C4: It probably does say quite a lot that the lecturer thinks in terms of domain etc. and not overtly about graphs.
C5: I would be drawn towards a low-tech approach, roughly draw them and insert them on the side
C6: but I wouldn't find them necessary for answering this question.
C7: I would like the students though to carry the graphs of all these functions in their heads straightaway and have them immediately available.
C8: But then again there are less and more visual people and the more visual may think that a question like this can only be done by producing the graph, using it and then proving the claim formally.
C9: Graphs are good ways to communicate mathematical thought and I do not wish to underplay that at all.

The correspondence between transcript clauses, uttered by the interview participants, and clauses C1-C9, uttered by M, is shown in Table 1:

Transcript	M_S	M_S M_{I1} M_{I2}	M_{M1} M_{I3}	M_{P1}	M_{T2}	M_{T2}	M_{T2}	M_{P2}	M_{P2} M_{M3}
Clause	C1	C2	C3	C4	C5	C6	C7	C8	C9

Table 1. An example of the correspondence between participant utterances in the transcript (first row) and clauses uttered by M (second row).

I note the following in relation to how the utterance of M came to be:

- Utterances M_J, RME_{11} and M_{M2} which highlight the difficulty with producing graphs for some of the functions in the problem sheet question (see the Appendix) are not included in this particular utterance of M. They are, however, consolidated into a later part of *Episode 4.3* that focuses on the challenge of having a clear and transparent perspective on what knowledge about functions the students can assume at this stage and how they can deploy this knowledge towards the construction of function graphs.
- RME_{22}, RME_{12} and M_{T1} highlight also that we cannot judge the lecturer for not including graphs in the suggested solutions as these are emphatically presented as

only *notes* on solutions in the document given to the students. The many and varied ways of presenting mathematical writing to students (also alluded at in M_{I3}) are also dealt with in other (in fact, numerous other) *Episodes*, but not in this utterance. This is a characteristic example of how interviewee utterances across the interviews have been exported/imported in the composition of *Episodes*, thus strengthening the thematic tightness of these *Episodes*.

- The divergence of views between M_T and M_P, evident in M_{T2} and M_{P2}, is reflected in the 'But then again…' part of C8. This is a characteristic example of how M often expresses a range of views. The view that appears as the ultimate one in the dialogues in Nardi (2008) is typically what I judged as closer to a consensus amongst the participants (or lack of, in the cases where no such consensus concludes the *Episodes*, as, for example, is the case for several *Episodes* in Chapter 8).

With the outline presented in this section I also aim to indicate how the *re-storying* process was carried out with Bruner's (1991) ten characteristics of narrative in mind. For example, in M's utterance used as an example in this section, there are elements of *diachronicity* (M discusses pedagogical approaches to the inclusion of graphs that range across several phases of students' learning about functions), *intentional state entailment* (M clearly states beliefs about the value of visualisation in doing and learning mathematics), *canonicity and breach* (M delineates how this pedagogical approach assimilates the diversity of needs from learners who are less or more visual in their preferences); and, *normativeness* (M clearly states what he wishes to see in how the students approach their learning about functions). In closing, I consider the *re-storying* approach as a key component of a type of research that has the potential to enhance rapprochement between two communities – mathematicians and mathematics educators – often separated by substantial epistemological and pragmatic differences.

6. *RE-STORYING* AS A VEHICLE FOR COMMUNITY RAPPROCHEMENT IN MATHEMATICS EDUCATION

The claim that I put forward in this paper is not one of superiority of the narrative approach of *re-storying* to other approaches (such as rigorous thematic analysis of interview data). Rather, the claim is that this particular form of generating insights into university mathematics pedagogy (*non-deficit*, *non-prescriptive*, *context-specific*, *example-centred* and *mathematically-focused*) addresses some of these differences and offers an alternative way in which the communication between the two communities can take place. A key feature, for example, in the dialogues in *Amongst Mathematicians* is that they are jargon-free, even though their construction is fundamentally driven by the mathematics education research findings cited in the footnotes that are present on almost every page. One of the pragmatic differences between the two communities cited in the literature quoted in Section 1 is the absence of a common language in which mathematicians and mathematics educators can discuss teaching and learning. The dialogues in *Amongst Mathematicians* are intended as a potent communicative tool: their constitutive elements are the mathematicians' insights into university mathematics pedagogy contributed over a lengthy period of elaborate discussions with mathematics educators, woven together with the mathematics educators' insights emerging out of their knowledge of the research literature in this field. In other words, I propose *re-storying* as a vehicle for community rapprochement achieved through generating and sharing research findings – the *substance* of research – in *forms* that reflect the fundamental principles and aims that underpin this research.

The two communities of mathematics and research in mathematics education – which intersect in at least one juncture, in the *joint enterprise* (Biza, Jaworski & Hemmi, 2014) of mathematics teaching at university level – need to meet, confer and generate negotiated, mutually acceptable perspectives more often (Artigue, 1998). Through a demonstration of the rich pedagogical canvas that is evident in the utterances of M, this emphatically evidence-based approach is intended not only as a contribution to their rapprochement, but also as a riposte to stereotypical views that see university mathematics teaching practitioners as non-reflective actors who rush through content-coverage in ways often insensitive to their students' needs, and who have no pedagogical ambition other than that related to success in examinations and audits. Simultaneously, it challenges presentations of mathematics education researchers as having a suspiciously loose commitment to the cause of mathematics, and whose irrelevant theorizing renders them incapable of 'connecting' with practitioners. The dialogues that came into being through the research design presented in this paper – of which the *re-storying* approach is a key component – are intended as an embodiment of these ripostes.

For example, the more discrete presence of RME in the interviews – and then, in symmetry, also in the dialogues – is intended to create a space in which M can showcase views on, and experiences of, university mathematics pedagogy in the reflective atmosphere of the group interviews. As noted above, the minimal presence of RME in the dialogues should not detract from their fundamental role in the choice of data samples to be discussed in the interviews and in the shaping of the themes in the *Episodes* (including the essential component of embedding the dialogues in the relevant literature through the footnotes in the text). It is therefore in the dialogues (and what the two communities can do, and have been doing, with them) that what Gutiérrez, Baquedano-López & Tejeda (1999) call a *third space* – "the particular discursive spaces in which alternative and competing discourses and positionings transform conflict and difference into rich zones of collaboration and learning" (p. 286-7) – is in action.

Originally conceived as a way to describe and contest elements of Vygotsky's Zone of Proximal Development (Gutiérrez *et al.*, ibid.), the remit of the *third space* construct has been expanded to accounts of "the concrete and material practices of a transformative learning environment" (Gutiérrez 2008, p. 148) and, recently, to accounts of transformative learning experiences at university level (Hernandez-Martinez, 2013). I contend that working with, and being exposed to, *novel* processes towards the generation and presentation of research findings, such as that of *re-storying*, supports the construction of such a *third space*. This is a space characterized by what Nolan (2010) captures neatly in his view of mathematics teacher education as contemporary *praxis:* 'Praxis seeks to create not a contentious dichotomy between theory and practice but instead a dialogic, dialectic relationship that highlights a continual interplay between them' (p. 726).

One of the objectives of the *re-storying* approach proposed here is what Pais (2013) describes as superseding the traditional macro/micro divide: overcoming this dichotomy to realise how the universal (macro) manifests itself in concrete situations and to acknowledge how the universal operates within the particular which, in return, colours its very universality and accounts for its efficiency. In tune with Pais, the *re-storying* approach attempts to capture what the universal (the claim, for example, that mathematics education research can provide quick-fix, water-tight pedagogical prescriptions) secretly excludes; and, to observe how epistemological belief and institutional practice/policy is enacted through the situation-specific, context-bounded utterances of individuals, all involved with mathematical pedagogy but who may come from different, but often crossing, disciplinary and institutional paths (Nardi, 2016, in press).

In the dialogues constructed out of the focus group interview data exemplified in this paper, knowledge (mostly about mathematical pedagogy) is relocated distinctly away from typical *mathematical* epistemologies but, even more crucially, as far away as possible from decontextualized pedagogical prescription. The proposition made here is that this new form of knowledge about mathematical pedagogy, co-constructed by members of two often separated communities (mathematicians and mathematics educators), is relocated to a novel *third space* which welcomes the *non-deficit*, *non-prescriptive*, *context-specific*, *example-centred* and *mathematically-focused* discourses that govern the production and communication of this knowledge.

REFERENCES

Andrews, M. (2006). Exploring cross-cultural boundaries. In D. J. Clandinin (Ed.), *Handbook of narrative inquiry: Mapping a methodology* (pp. 489-511). Thousand Oaks, CA: Sage.

Artigue, M. (1998). Research in mathematics education through the eyes of mathematicians. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a research domain: A search for identity* (pp. 477 - 490). Dordrecht / Boston / London: Kluwer Academic Publishers.

Barrett, M.S. & Stauffer, S.L. (2009). Narrative Inquiry: From Story to Method. In M. S. Barrett & S. L. Stauffer (eds.) *Narrative Inquiry in Music Education: Troubling certainty* (pp. 7-17). NY: Springer.

Bateson, M. C. (1994). *Peripheral visions: Learning along the way*. New York: Harper Collins.

Biza, I., Jaworski, B., & Hemmi, K. (2014). Communities in university mathematics. *Research in Mathematics Education 16*(2), 161-176.

Biza, I., Nardi, E., & Zachariades, T. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education 10*, 301–309.

Bruner, J. (1991). The narrative construction of reality. *Critical Inquiry, 18*(1), 1-21.

Burton, L. (2002). Children's mathematical narratives as learning stories. *European Early Childhood Education Research Journal 10*(2), 5-18.

Cantú, N. (2012). Getting There Cuando No Hay Camino (When There Is No Path): Paths to Discovery Testimonios by Chicanas in STEM. *Equity & Excellence in Education 45*(3), 472-487.

Carr, D. (1986). Time, narrative, and history. Bloomington: Indiana University Press.

Charmaz, K. (2005). Grounded theory: Objectivist and Constructivist Methods. N. K. Denzin & Y. S. Lincoln (eds) *Handbook of Qualitative Research (3rd ed.)* (pp. 509 – 535). New York: Sage.

Chase, S. E. (2011) Narrative Inquiry: Still a Field in the Making. N. K. Denzin & Y. S. Lincoln (eds) *Handbook of Qualitative Research (4th ed.)* (pp. 421–34). Thousand Oaks, CA: Sage Publications.

Clandinin, D.J. (Ed.) (2008). Handbook of narrative inquiry: Mapping a methodology. Thousand Oaks, CA: Sage Publications.

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. San Francisco, CA: Jossey-Bass.

Clandinin, D. J., Pushor, D., & Orr, A. M. (2007). Navigating sites for narrative inquiry. *Journal of Teacher Education, 58*(1), 21-35.

Conle, C. (2000). Narrative Inquiry: research tool and medium for professional development. *European Journal of Teacher Education 23*(1), 49-63.

Connelly, F. M., & Clandinin, D. J. (1990). Stories of experience and narrative inquiry. *Educational Researcher, 19*(5), 2-14.

Connelly, F. M., & Clandinin, D. J. (2000). Narrative understandings of teacher knowledge. *Journal of Curriculum Supervision, 14,* 315-331.

Connelly, F. M., & Clandinin, D. J. (2005). Narrative inquiry. In J. Green, G. Camilli, & P. Elmore (Eds.) Handbook of complementary methods in educational research (pp. 477-489). Washington, DC: American Educational Research Association.

Darragh, L. (2013) Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education 15*(3), 215-229.

Denzin, N. K., & Lincoln, Y.S. (Eds.). (2011). *Handbook of qualitative research (4th ed.)*. Thousand Oaks, CA: Sage Publications.

Frayn, M. (1998). *Copenhagen*. London: Methuen Drama.

Frost, J.H. (2010). Looking through the lens of a teacher’s life: The power of prototypical stories in understanding teachers’ instructional decisions in mathematics. *Teaching and Teacher Education 26*, 225-233.

Gutiérrez, K.D. (2008). Developing a sociocritical literacy in the third space. *Reading Research Quarterly 43*(2), 148-164.

Gutiérrez, K.D., Baquedano-López, P. & Tejeda, C. (1999). Rethinking diversity: Hybridity and hybrid language practices in the third space. *Mind, Culture, and Activity 6*(4), 286-303.

Healy, L. & Sinclair, N. (2007). If this is our mathematics, what are our stories? *International Journal of Computers in Mathematical Learning* 12, 3-21.

Hernandez-Martinez, P. (2013). Teaching mathematics to engineers: Modelling, collaborative learning, engagement and accountability in a Third Space. www.esri.mmu.ac.uk/mect2/papers_13/hernandez.pdf

Kemmis, S. (2010). Research for praxis: knowing doing. *Pedagogy, Culture and Society 18*(1), 9-27.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge, UK: Cambridge University Press.

Moreau, M.P, Mendick, H. & Epstein, D. (2010). Constructions of mathematicians in popular culture and learners' narratives: a study of mathematical and non-mathematical subjectivities. *Cambridge Journal of Education 40*(1), 25-38.

Nardi, E. (2008). *Amongst Mathematicians: Teaching and learning mathematics at university level*. USA: Springer.

Nardi, E. (2016, in press). Mathematics and Mathematics Education: A story of paths just crossing or of meeting at a vanishing point? *Mathematics Today* (Special Issue: Windows on Advanced Mathematics).

Nardi, E. & Iannone, P. (2003) Mathematicians On Concept Image Construction: Single 'Landscape' Vs 'Your Own Tailor-Made Brain Version'. *Proceedings of the 27th Annual Conference of the International Group for Psychology in Mathematics Education III* (pp. 365-372). Honolulu: USA.

Nardi, E., Ryve, A., Stadler, E., & Viirman, O. (2014) Commognitive analyses of the learning and teaching of mathematics at university level: the case of discursive shifts in the study of Calculus. *Research in Mathematics Education 16*(2), 182-198.

Nolan, K. (2010). Praxis. In A. Mills, G. Durepos, & E. Wiebe (Eds.), *SAGE Encyclopedia of Case Study Research* (Vol. 2, pp. 725-727). UK: Sage

Ollerenshaw, J. A., & Creswell, J. W. (2002). Narrative research: A comparison of two restorying data analysis approaches. *Qualitative Inquiry 8*(3), 329 – 347.

Pais, A. (2013). An ideology critique of the use-value of mathematics. *Educational Studies in Mathematics 84*, 15 – 34.

Ricoeur, P . (1984, 1985, 1988). *Time and narrative (Vols. I-III).* Chicago University Press.

Schafer, R. (1981). Narration in the psychoanalytic dialogue. W. J. T. Mitchell (Ed.) *On narrative* (pp. 25-50). Chicago: University of Chicago Press.

Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourse, and mathematizing*. New York: Cambridge University Press.

Solomon, Y. (2012). Finding a voice? Narrating the female self in mathematics. *Educational Studies in Mathematics 80*, 171–183.

Stoppard, T. (1993). *Arcadia*. London: Faber and Faber.

Tall, D. O. and Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*, 151-169.

Webster, L. and Mertova. P. (2007). *Using Narrative Inquiry as a Research Method.* UK: Routledge.

APPENDIX

P. 8-9 of the 16-page data sample distributed to the interviewees prior to one of the six interviews carried out as a series over a whole academic year in one participating mathematics department

Page 8: the mathematics question (Week 4, introductory Y1 Calculus course) followed by the lecturer's suggested response (distributed to students after the completion of their coursework)

For each of the following functions $\mathbb{R} \to \mathbb{R}$ decide whether it is one-to-one, onto (or both, or neither). Give brief explanations for your answers.

(i) $f_1(x) = \sin x + \cos x$

(ii) $f_2(x) = 7x + 3$

(iii) $f_3(x) = e^x$

(iv) $f_4(x) = x^3$

(v) $f_5(x) = \frac{x}{1+x^2}$

Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is onto but not one-to-one.

A problem sheet question

(i) As $\sin x \leq 1$ and $\cos x \leq 1$ for $x \in \mathbb{R}$, we have $f_1(x) \leq 2\ \forall x \in \mathbb{R}$. Thus f_1 is not onto. Also, $f_1(0) = f_1(2\pi)$, so f_1 is not one-to one.

(ii) For every $y \in \mathbb{R}$ there is unique $x \in \mathbb{R}$ with $f_2(x) = y$, namely $x = \frac{1}{7}(y-3)$. Thus f_2 is one-to-one and onto.

(iii) Not onto (as $e^x > 0$ for all $x \in \mathbb{R}$): one-to-one (if $y \in \mathbb{R}$ the only real solution to $e^x = y$ is $x = lny$).

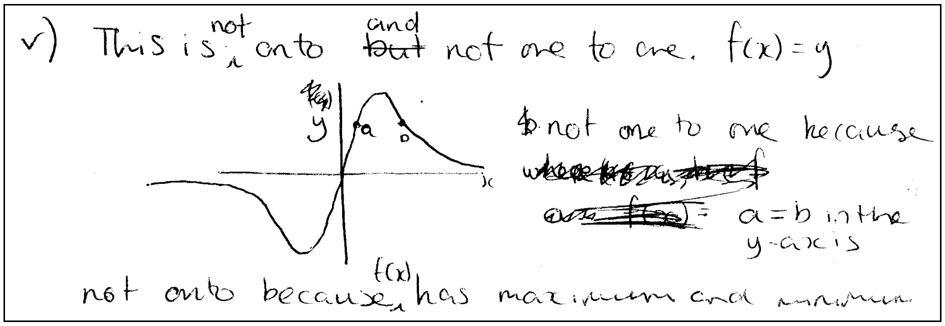
(iv) One-to-one and onto (a bijection): any real number has a unique real cube root.

(v) Neither one-to-one nor onto: $f_5(\frac{1}{2}) = f_5(2) = \frac{2}{5}$, so not one-to-one. Also, $f_5(x) \leq 1$ (as this is equivalent to $x \leq 1 + x^2$, and $1 - x + x^2 = (x - \frac{1}{2})^2 + \frac{3}{4} \geq 0$.

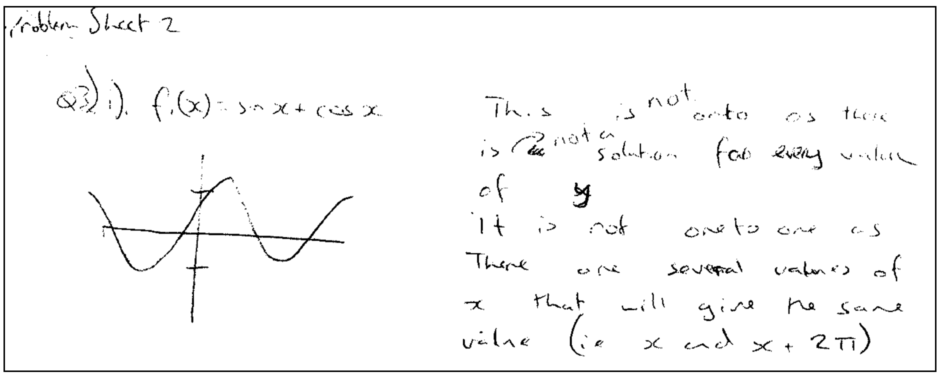
Remark: You might like to think about what bits of calculus can be used to justify more fully the fact that the functions in (iii) and (iv) are one-to-one.
Last part: $f(x) = x(x-1)(x+1)$ is onto but not one-to-one.

The lecturer's suggested response

Page 9: two typical student responses to above problem sheet question



Student WD



Student LW

This was followed by some *Examples of issues to consider:*

- Student WD's response relies completely on the observation of the graph of the function (see points *a* and *b* on a parallel to the *x*-axis). His answers are correct but lack formal justification. What is the implication then of relying on the diagram for the students' acquisition of formal reasoning skills?
- Student LW's response, where the graph is inaccurate, represents the potential risks within the practice of relying exclusively on the diagrams. Like the shift from idiomatic use of symbolic language to a conventional one in [prior example in the data sample], here the shift seems to be from relying on (potentially misleading) visual evidence to employing visual evidence as a tool that supports understanding. How can teaching facilitate this shift?

Note: This is an excerpt from *Episode 4.3* (Nardi, 2008, p. 139-150).