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# Particles and Propagators in Relativistic Thermo Field Theory

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Abstract

Based on a new spectral representation of thermal correlation functions, a general characterization of stable particles in relativistic thermo field theory is given. Such particles manifest themselves by specific discrete contributions in the spectral functions and can thus be identified unambiguously.

## 1 Introduction

The characterization of the particle content of thermal equilibrium states is a longstanding conceptual problem in the general framework of relativistic thermo field theory [1,2]. It appears for example in quantum chromodynamics where one would like to understand in which precise sense the constituents of a quark gluon plasma may be viewed as particles.

The standard description of particles by superpositions of waves with a sharp dispersion law is not adequate if dissipative effects impede their propagation through a thermal background. Then particles do not manifest themselves by discrete mass shell singularities in the Fourier transforms of the underlying correlation functions, as was first pointed out by Narnhofer, Requardt and Thirring. In fact, any discrete mass shell contribution in these functions corresponds to a particle which does not scatter [3]. In the presence of interaction it is therefore difficult to discriminate particles from other modes. We note that we use here the term particle both for the constituents and the respective holes in an equilibrium state.

In an investigation of this problem Landsman [4] proposed to interpret the particle structure of interacting thermo field theories in terms of elementary unstable entities described by generalized free fields. Yet this approach does not really help to extract the particle content from a given correlation function. Moreover, the physical idea that particles in a thermal state must always be regarded as unstable is not very plausible. As we shall discuss, the absence of mass shell singularities in the correlation functions may merely be an expression of the fact that a particle which is launched into a thermal state disappears after some time in the maze of other particles present. One simply loses track of it. This mechanism is very different from the decay of a resonance, say, into stable subsystems.

In the present note we reconsider this problem and outline a method which allows one to extract from a given correlation function certain specific contributions which can be attributed to particles. To this end we first introduce a class of correlation functions which model the propagation of a particle of mass $m$ through a thermal background. The second ingredient is the information [5] that all two point correlation functions in relativistic thermo field theory can be represented as direct integrals (with respect to the mass $m$) of our model functions. This representation incorporates all general features of relativistic thermo field theory and hence is more specific than previous results, cf. [1,2,4]. Making use of these facts we then propose to identify stable particles by those contributions in the underlying correlation functions which appear in our integral representation with discrete weight $m$. This procedure is completely analogous to the determination of the particle content in vacuum quantum field theory.

Our results are based on the following general assumptions. We consider any local, hermitian and (for simplicity) scalar field $\phi(x)$ on Minkowski space and any thermal equilibrium state $\langle \cdot \rangle_\beta$ which is homogenous and isotropic. Hence there holds $\langle \phi(y)\,\phi(x)\rangle_\beta = W_\beta(x-y)$, where $W_\beta(x)$ is the Fourier transform of a positive density $\widetilde{W}_\beta(p)$ (measure),

$$W_\beta(x) = (2\pi)^{-2} \int d^4p\, \widetilde{W}_\beta(p)\, e^{ipx}. \tag{1}$$

The condition of locality implies that for spacelike $x$

$$W_\beta(x) = W_\beta(-x). \tag{2}$$

The fact that $\langle \cdot \rangle_\beta$ describes an equilibrium state at inverse temperature $\beta$ is encoded in the Kubo–Martin–Schwinger (KMS) condition. According to that condition, the distribution $W_\beta(x)$ can be analytically continued in the time variable $x_0$ into the strip $0 < Im\, z_0 < \beta$, and its boundary value at the upper rim of this strip is given by

$$\lim_{y_0 \nearrow \beta} W_\beta(x + i(y_0, \mathbf{0})) = W_\beta(-x). \tag{3}$$

The KMS condition can be established [6] if the state $\langle \cdot \rangle_\beta$ is the thermodynamic limit of a Gibbs canonical ensemble in a finite volume $V$ with correlation function

$$W_{\beta,V}(x) = Z^{-1} \cdot \mathrm{Tr}\, e^{-\beta H_V}\, \phi(0)\phi(x), \tag{4}$$



where $H_V$ denotes the finite volume Hamiltonian with appropriate boundary conditions and $Z$ the respective partition function.

We note that, in relativistic thermo field theory, relation (4) seems to imply much stronger analyticity properties of $W_\beta(x)$ than those imposed by the KMS condition. In fact in a relativistic theory the finite volume energy momentum operators $(H_V, \mathbf{P}_V)$ in general have joint spectrum in the forward light cone. The correlation functions $W_{\beta,V}(x)$ thus have an analytic continuation in $x$ into the larger domain $\{z \in \mathbb{C}^4 : \mathrm{Im}\, z \in V_+ \cap ((\beta, \mathbf{0}) + V_-)\}$, where $V_\pm$ denotes the forward, respectively backward lightcone. Now, apart from phase transition points, the local properties of the correlation functions $W_{\beta,V}(x)$ should not be sensitive to the size of the volume $V$ and we therefore expect that these analyticity properties persist in the thermodynamic limit in generic situations. If this happens to be the case, we say that $W_\beta(x)$ satisfies the *relativistic KMS condition*. A similar remark applies to the grand canonical ensembles, though in that case the cones $V_\pm$ in the above domain have to be replaced by smaller ones, depending on the chemical potential.

Finally we assume that the state $\langle \cdot \rangle_\beta$ describes a pure phase and thus has the clustering property. This implies, after subtracting from $\phi(x)$ some multiple of the identity if necessary, that

$$\lim_{y_0 \to \pm\infty} W_\beta(x + (y_0, \mathbf{0})) = 0, \tag{5}$$

hence $\widetilde{W}_\beta(p)$ does not have any discrete ($\delta$-function) contribution at $p_0 = 0$. We emphasize that we are dealing here with the correlation functions of the underlying fields. But our results can easily be extended to the time ordered, retarded and advanced Green functions as well as to the doubled field formalism used in thermo field dynamics [1].

## 2 Dissipative Propagation

Since plane waves in general do not describe the propagation of particles in a thermal state, we introduce here another basis of functions which take into account the dissipative effects of a thermal background. These functions have a particle interpretation and are sufficient for the analysis and synthesis of arbitrary correlation functions $W_\beta(x)$.

To explain the general idea, let us first consider the two point correlation function describing the free propagation of a particle of mass $m$ in an equilibrium state of inverse temperature $\beta$, which is given by

$$W_\beta^{(0)}(x;m) = (2\pi)^{-3} \int d^4p\, \varepsilon(p_0)\delta(p^2 - m^2)\,(1 - e^{-\beta p_0})^{-1}\, e^{ipx}. \tag{6}$$

This distribution satisfies all general requirements mentioned in the Introduction, including the relativistic KMS conditon. Moreover, it is meaningful to interpret its positive, respectively negative frequency parts as probability amplitudes for finding at the spacetime point $x$ a particle (hole) of mass $m$ which has been created at 0.

In the presence of interaction, this particle will collide with a certain probability with other constituents of the state. The resulting excitations can in general no longer be interpreted in terms of a single particle, they contribute primarily to higher correlation functions describing multiple excitations. But it will also happen that the particle manages to get to the point $x$ without intermediate collisions, though the probability for finding it there will in general be smaller than in the free case. We therefore make for the effective probability amplitude the Ansatz

$$W_\beta^{(D)}(x;m) = D(x) \cdot W_\beta^{(0)}(x;m), \tag{7}$$

where $D(x)$ is a damping factor which is regular (locally analytic) in $x$. This Ansatz subsumes the expected properties of a stable particle in a thermal state: it may disappear in the background because of collisions whilst the momentum space density $\widetilde{W}_\beta^{(D)}(p;m)$ still exhibits spectral properties resembling a particle structure. The latter point is accomplished by our regularity assumption which implies that the density $\widetilde{W}_\beta^{(D)}(p;m)$ is



concentrated about the mass shell $p^2 = m^2$ due to the support properties of $\widetilde{W}_\beta^{(0)}(p;m)$ and the rapid decay of the Fourier transform $\widetilde{D}(p)$.

Since we want to interpret $W_\beta^{(D)}(x;m)$ as a correlation function, the damping factor $D(x)$ has to comply with further constraints. The condition of locality requires that $D(x) = D(-x)$ for spacelike $x$ which, because of local analyticity, is only possible if $D(x)$ is symmetric in $x$. These symmetry properties and the KMS condition entail that $D(x)$ has to be entire analytic and periodic in $x_0$ with period $i\beta$, as well as polynomially bounded because of temperedness. Hence $D(x)$ has to be constant in $x_0$. We will therefore omit in the following the time variable in the argument of the damping factor. Another constraint arises from the fact that the density $\widetilde{W}_\beta^{(D)}(p;m)$ has to be positive. The ensuing condition on $D(\mathbf{x})$ is not particularly instructive and we therefore omit it here. But we note that the condition is satisfied if the Fourier transform $\widetilde{D}(\mathbf{p})$ is positive.

If one imposes on $W_\beta^{(D)}(x;m)$ the relativistic KMS condition, the damping factor $D(\mathbf{x})$ has to be analytic in the domain $\{\mathbf{z} \in \mathbb{C}^3 : |\mathrm{Im}\,\mathbf{z}| < \beta/2\}$. Then $\widetilde{D}(\mathbf{p})$ decreases for large $|\mathbf{p}|$ like $\exp(-\beta|\mathbf{p}|/2)$ and consequently the density $\widetilde{W}_\beta^{(D)}(p;m)$ is concentrated in a neighbourhood of the mass shell fixed by $\beta^{-1}$. Hence at low temperatures one obtains a rather sharp dispersion law for the particle, as heuristically expected in the canonical ensemble.

A simple example exhibiting all desired features of a damping factor is provided by the function $D(\mathbf{x}) = c \cdot e^{-(|\mathbf{x}|^2+\beta^2/4)^{1/2}/\lambda}$, where $\lambda$ may be interpreted as the mean free path of the particle in the underlying equilibrium state and $c$ may be fixed by the normalization condition $D(\mathbf{0}) = 1$. The corresponding density $\widetilde{W}_\beta^{(D)}(p;m)$ can be calculated explicitly. For large $\beta/\lambda$ it decreases "off mass shell" like a Gaussian with variance proportional to $1/\lambda\beta$, hence at low temperatures one obtains the expected sharp dispersion law for the particle. If $\beta/\lambda$ is small one can extract from the density a dominant contribution which, for $|\mathbf{p}|$ large compared to $\lambda^{-1}$, is of Breit-Wigner type with poles at $p_0 = \pm\epsilon \pm i|\mathbf{p}|/\lambda\epsilon$, where $\epsilon = (|\mathbf{p}|^2 + m^2)^{1/2}$. The restrictions on $|\mathbf{p}|$

imply that the de Broglie wave length of the particle is small compared to its mean free path, which is a necessary prerequisite for a particle interpretation. Because of the pole structure one can then attribute to a particle with velocity $v = |\mathbf{p}|/\epsilon$ a "mean lifetime" $\lambda/v$. This result is consistent with our interpretation of $\lambda$ since within the time $\lambda/v$ the particle covers the distance $\lambda$ and likely will hit another particle, thereby producing a multiple excitation which does not contribute to our correlation function. In this specific sense the particle then ceases to exist.

Our heuristic considerations have led us to a class of correlation functions $W_\beta^{(D)}(x;m)$ which model the dissipative propagation of a stable particle through a thermal state. Since the damping factors do not depend on time the functions $W_\beta^{(D)}(x;m)$ have an energy gap $\pm m$ which allows one to recover from them the mass of the underlying particle. The transition probabilities for the states of the particle (hole) which, at a given time, are localized in small regions (compared to the mean free path) can consistently be defined by

$$_\pm\langle f, x_0 | g, y_0\rangle_\pm \;=\; \int d^3x \int d^3y\, \overline{f(\mathbf{x})} g(\mathbf{y})\, W_{\beta,\pm}^{(D)}(y-x;m). \tag{8}$$

Here $f(\mathbf{x})$ and $g(\mathbf{y})$ are wave functions with support in the respective regions and the subindex $\pm$ indicates the positive, respectively negative frequency part of the correlation function.

One may proceed from any one of the correlation functions $W_\beta^{(D)}(x;m)$ to a fully consistent relativistic (non–Lagrangean) thermo field theory by putting all higher truncated functions equal to zero. In these theories the dissipative effects of a thermal background are taken into account, but there is no direct interaction between the particles. Such theories may therefore be suitable for the description of the asymptotic motion of many particle excitations of a thermal state, in analogy to the free field theories in the vacuum case.



# 3 Representation of Correlation Functions

General representations of correlation functions which take into account the KMS condition are well known, cf. [1,2,4]. Yet in none of these representations the condition of locality has been incorporated. We therefore present here a representation which exhibits explicitly this fundamental feature of a relativistic theory: Any correlation function $W_\beta(x)$ satisfying the general conditions in the Introduction can be represented in the form [5]

$$W_\beta(x) \;=\; \int_0^\infty dm\, D_\beta(\mathbf{x};m)\cdot W_\beta^{(0)}(x;m), \tag{9}$$

where the "damping factor" $D_\beta(\mathbf{x};m)$ is a distribution in $\mathbf{x}$ and $m$ which is symmetric in $\mathbf{x}$. Conversely, given any such distribution, relation (9) defines a correlation function $W_\beta(x)$ which satisfies the KMS condition and the condition of locality. The precise definition of the formal expression on the right hand side of (9) requires some care, it is to be understood in the sense of distributions [5]. But if the given correlation function $W_\beta(x)$ satisfies the relativistic KMS condition, then $D_\beta(\mathbf{x};m)$ can be shown to be regular in $\mathbf{x}$ and to have an analytic continuation into the domain $\{\mathbf{z} \in \mathbb{C}^3 : |\mathrm{Im}\,\mathbf{z}| < \beta/2\}$. It is of interest here that relation (9) can be inverted. One finds for $D_\beta(\mathbf{x};m)$, $m > 0$, the formal expression

$$D_\beta(\mathbf{x};m) = -2\pi i\,\frac{\partial}{\partial m}\int dx_0\, x_0 J_0(m\sqrt{x^2})\cdot C_\beta(x) \tag{10}$$

where $C_\beta(x) = W_\beta(x) - W_\beta(-x)$ is the commutator function and $J_0$ the zeroth order Bessel function of first kind. Again, this expression is well defined in the sense of distributions. The proof of these results, given in [5], is plagued by technicalities in view of the singular nature of the quantities involved. But the basic idea is quite simple and in fact a classic in vacuum quantum field theory. One starts from the well known fact that the KMS condition implies that

$$(1-e^{-\beta p_0})\cdot \widetilde{W}_\beta(p) \;=\; \widetilde{C}_\beta(p). \tag{11}$$

Because of locality $C_\beta(x)$ has support in $\overline{V}_+ \cup \overline{V}_-$ and, as was shown by Dyson [7], any such distribution can be represented in terms of solutions of the six dimensional wave equation. Plugging that representation into equation (11) and dividing by $(1 - e^{-\beta p_0})$, which is possible without ambiguities since $\widetilde{W}_\beta(p)$ has no disrete contribution at $p_0 = 0$, one arrives in a straightforward manner at the desired result.

It would be of interest to have better control on the regularity properties of $D_\beta(\mathbf{x}; m)$ in $m$, but to this effect more specific information on the correlation functions $W_\beta(x)$ is needed. In order to get some idea which properties of $D_\beta(\mathbf{x}; m)$ one might expect, it is instructive to proceed to the limit of zero temperature and density. In this limit the representation (9) turns into the Källén–Lehmann representation of the vacuum theory, i.e. the correlation function $W_\beta^{(0)}(x; m)$ becomes the positive frequency part of the Pauli–Jordan distribution and, for the case of a scalar field considered here, $D_\beta(\mathbf{x}; m)$ becomes independent of $\mathbf{x}$ because of Lorentz invariance. Making use of the condition of positivity this distribution can then be shown to be a positive density. Although $D_\beta(\mathbf{x}; m)$ will change substantially if one proceeds to finite temperatures, one may expect that its regularity properties persist, i.e. the worst singularities in $m$ should be $\delta$-functions. As was already pointed out $D_\beta(\mathbf{x}; m)$ is regular in $\mathbf{x}$ if there holds the relativistic KMS condition.

We mention as an aside that relation (9) gives rise to an interesting relation between the damping factors $D_\beta(\mathbf{x}; m)$ for arbitrary values of $\beta$. Assuming for a moment that the field $\phi(x)$ satisfies canonical commutation relations we would infer that

$$\int_0^\infty dm D_\beta(0; m) = 1. \tag{12}$$

In general this integral will not exist because of ultraviolet problems, but since it governs the short distance behaviour of the field $\phi(x)$ it has to exhibit the same divergent behaviour for all temperatures. This is a consequence of the fact that the Hilbert space representatives of the field $\phi(x)$ induced by KMS states of different temperatures will in general be locally unitarily equivalent (they arise from the same finite volume theory) and therefore have the same short distance behaviour. Hence in an asymptotically free



theory, where the above integral diverges only logarithmically, one may expect that the difference $D_{\beta'}(0; m) - D_{\beta''}(0; m)$ integrates to zero for arbitrary $\beta'$ and $\beta''$. This relation could be used for the field renormalization in KMS states, once the renormalization has been fixed in the vacuum theory. Such a scheme seems natural since the properties of equilibrium states ought to be completely determined if the vacuum theory is given [8].

## 4 Particle Analysis

In order to be able to identify in a given correlation function those contributions which can be attributed to a single particle one needs some a priori concept of particle within the theoretical setting. In vacuum theory such a concept has been provided by Wigner [9] who proposed to identify the states of a particle with vectors in some irreducible representation of the Poincaré group. This characterization is independent of the dynamics and provides a powerful tool for the particle analysis in vacuum theory, even though it has certain limitations in theories with long range forces.

In the case of thermo field theory the situation is more complicated. There one deals with states consisting of a maze of particles interacting with each other, and it is clear from the outset that it is not possible to characterize particles in such an environment in general mathematical terms. The best one can hope for is to identify certain characteristic features of particles in a thermal state which may then be taken as their defining properties. This is the strategy which we adopt here.

In the preceding discussion we followed up the physical idea that stable particles manifest themselves by certain specific singly localized excitations of the underlying equilibrium state and consequently should contribute already to the two-point correlation function. It was then natural to ask which of these functions are suitable for their description. Starting from the free correlation function which has an unquestionable particle interpretation we were led to such a class by taking into account dissipative effects through a damping factor. It turned out that in relativistic thermo field theory this Ansatz is only consistent if the damping factor does not depend on time. This observation is crucial, for it implies that one can reconstruct from the corresponding correlation functions all relevant data of the particle. The large ambiguity left in the form of the damping factor corresponds to the multifarious ways in which a particle can interact with a thermal background.

In a second step we have determined the general form of the two point correlation functions in relativistic thermo field theory without reference to particles. It turned out that these functions can be represented as direct integrals of functions of the type found before. This result shows on one hand that our Ansatz modelling dissipative propagation is general enough to cover all cases of interest. On the other hand it prepares the ground for our ultimate goal, the identification of stable particles in a thermal state.

At this point we can no longer ignore the fact (as we did in our model of dissipative propagation) that by acting with a field $\phi(x)$ on an equilibrium state one will in general not only produce excitations which may be interpreted as particles. With a certain probability one will also create metastable bound states, pairs, and other composite excitations. These excitations will not only have to endure the same dissipative effects as a particle, described by the damping factors, but they will disintegrate into multiply localized subsystems also without external impact. Hence the probability to recover at a spacetime point $x$ a composite excitation which has been created at 0 will be substantially smaller than for a particle.

This qualitatively different behaviour ought to find its mathematical expression in the fact that composite excitations contribute only to the continuous part of the integral representation (9), just as in vacuum quantum field theory. The corresponding amplitudes $D_\beta(\mathbf{x};m)\cdot W_\beta^{(0)}(x;m)$ then give, for fixed mass $m$, a negligible contribution to the observed probabilities. One could put a countable number of them equal to zero without changing $W_\beta(x)$. It is only the totality of these amplitudes which, in view of the many possibilities of forming composite excitations, adds up to a non trivial result.

In contrast, it is a characteristic feature of particles that they stay singly localized



unless they interact with some obstacle. The corresponding "loss" of particles has already been taken into account by the damping factor, hence stable particles ought to manifest themselves by non negligible contributions in (9). The integral representation then contains discrete parts of the form $\delta(m-m_0)\,W_\beta^{(D)}(x;m_0)$, where $m_0$ is the mass of the respective particle and $W_\beta^{(D)}(x;m_0)$ one of our model functions. Whether a given correlation function $W_\beta(x)$ contains such a contribution can be checked with the help of relation (10). Thus by this method one can identify within our framework the particle content of equilibrium states. We summarize these considerations in the form of a criterion which we propose for the characterization of stable particles in thermo field theory.

*Criterion: Any discrete part in the decomposition (9) of a correlation function corresponds to a particle with sharp mass $m$ which is stable with respect to perturbations of the underlying equilibrium state. The transition probabilities for the states of this particle (respectively hole) are fixed by the corresponding contribution $W_\beta^{(D)}(x;m)$ which can be recovered from the correlation function with the help of relation (10).*

We have stated this criterion for scalar fields. Analogous formulations, based on the representations of the respective correlation functions, can be given for arbitrary Bose and Fermi fields. Moreover, in view of the fact that the time dependence in the representation (9) is contained in the free correlation functions it is straightforward to reformulate the criterion in terms of the Green functions, where it may be more convenient for concrete applications.

We conclude this note with two remarks on further aspects of our analysis. First we note that discrete contributions in the representation (9) with mass $m>0$ can be distinguished from continuous ones by their different behaviour in time: they give, for fixed $\mathbf{x}$, rise to an asymptotic behaviour like $x_0^{-3/2}$, whereas continuous contributions lead to a more rapid decay. This fact might be of interest for the formulation of a collision theory in thermo field theory which, in view of the ubiquitous dissipative effects, will look different from the familiar scheme in vacuum theory, cf. [3]. We hope to return to this problem elsewhere.

Our second remark refers to the appearance of quasiparticles in thermal states, such as phonons or plasmons. These particle-like structures arise from cooperative effects of the constituents of a state and are thus of a somewhat different nature. But it is tempting to use for their characterization similar methods as applied here to particles. To this end it would be necessary to have some a priori information about the form of their respective dispersion laws. One could then specify a corresponding family of free correlation functions and try to take into account dissipative effects by damping factors. If the resulting basis of functions turns out to be sufficient to write down a resolution of all correlation functions of interest, similar to relation (9), one could characterize the quasiparticles in the same way as particles.



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