SINGULAR INTEGRAL EQUATIONS

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SINGULAR INTEGRAL EQUATIONS

Boundary problems of functions theory and their applications to mathematical physics

Revised translation from the Russian, edited by

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ANNOTATION

This monograph by N. I. Muskhelishvili systematically acquaints the reader with the mathematical apparatus of Cauchy type integrals and singular integral equations, in the study of which the author and his students took active interest. A considerable part of the book is devoted to applications to the solution of numerous problems of potential theory, the theory of elasticity and other sections of mathematical physics.

The book is intended for postgraduates and students of advanced courses of the physico-mathematical faculties, and likewise for research engineers.

EDITOR'S PREFACE

In preparing this translation for publication certain minor modifications and additions have been introduced into the original Russian text, in order to increase its readibility and usefulness. Thus, instead of the first person, the third person has been used throughout; wherever possible footnotes have been included with the main text. The chapters and their subsections of the Russian edition have been renamed parts and chapters respectively and the last have been numbered consecutively.

An authors and subject index has been added. In particular, the former has been combined with the list of references of the original text, in order to enable the reader to find quickly all information on any one reference in which he may be especially interested. This has been considered most important with a view to the difficulties experienced outside Russia in obtaining references, published in that country.

Russian names have been printed in Russian letters in the authors index, in order to overcome any possible confusion arising from transliteration.

Zürich. J. R. M. Radok

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PREFACE

This book is intended for an extensive field of readers, in particular for those interested in applications to the theory of elasticity, hydromechanics and other branches of mathematical physics. The book is accessible to those acquainted with the basic theory of functions of a complex variable and the theory of Fredholm integral equations. To facilitate the reading of the book, theorems, the method of proof of which is not of essential independent interest, have been printed in italics, so that the proofs may be omitted without affecting an understanding of the nature of the matter. In addition, wherever possible, the Parts and their chapters devoted to different applications have been made independent of one another. It is hoped that the methods studied in this book may be effectively employed in the solution of many problems of an applied character. Some simple applications to potential theory, the theory of elasticity and hydromechanics are given in this book.

The idea of writing this book resulted from the Author's lectures in a seminar at the Mathematical Institute, Tiflis, in 1940—1942. Under the influence of a series of results obtained by members of the seminar (chiefly due to the excellent work of I. N. Vekua) the range of problems which the Author proposed to examine was essentially altered; the Author may state with much satisfaction that a large proportion of the contents of this book must be considered as the result of the collective work of the young members of the Tiflis Mathematical Institute of the Academy of Sciences of the Georgian S.S.R. together with I. N. Vekua and the Author himself.

Tiflis, Autumn 1944 N. I. Muskhelishvili

In adding the last corrections to the book the Author desires to use this opportunity to express his deep gratitude to the publishing house which always willingly complied with the Author's suggestions. The difficult and responsible task of proof-reading was considerably eased for the Author by the extraordinary kindness of L. I. Bokshitski on the staff of the press, to whom he extends his sincere gratitude just as to all the other members of the staff of the 16th Printing Establishment of the State publishers for their prompt and efficient work.

Moscow, Autumn 1945 N. I. M.

INTRODUCTION

In recent years the theory of singular integral equations has assumed increasing importance in applied problems.

In this book only one-dimensional (i.e., where the range of integration is one-dimensional, i.e., a line) singular equations involving Cauchy principal values will be examined, since the theory of multi-dimensional equations of corresponding form is still far from completion. (Some references to the literature dealing with the latter are given in § 59).

The fundamentals of the theory of one-dimensional singular integral equations of the type described were included in the work of Poincaré and Hilbert, almost directly after the development of the classical theory of integral equations by Fredholm. However, the theory of singular integral equations did not receive the attention of mathematicians for some time. On the other hand many problems of an applied character naturally reduced to singular equations, e.g. problems of the theory of elasticity, etc.; thus often in practice these equations were arrived at by "ordinary methods" and this did not always lead to satisfactory results.

However, during recent years, the theory of one-dimensional singular integral equations has advanced considerably and it can now be presented in a finished form.

This theory appears to be particularly simple and effective, if the solution of a boundary problem of the theory of functions of a complex variable, to be called the Hilbert problem, is considered. Therefore the theory of singular equations is here closely linked with the above boundary problem. The solution of the latter will be used for the development of the theory of singular equations; afterwards this theory will be applied to the solution of other more complicated boundary problems, in particular, to problems encountered in potential theory, the theory of elasticity and in hydromechanics.

Having in mind the implications for different problems of mathematical physics, some restrictions will be imposed upon the unknown and the given functions appearing in the integral equations under consideration or in the boundary conditions of the problems considered, which will largely simplify the investigation, but not affect the final theory.

The fundamental tools of the investigation are Cauchy integrals, the elementary theory of which is given in Part I together with a number of direct simpler applications.

As regards the contents of the remaining Parts nothing will be said here, since a preliminary idea can be gained by means of the sufficiently detailed list of contents at the beginning of the book.