Maximum Entropy and Bayesian Methods

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# Maximum Entropy and Bayesian Methods

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edited by

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To the ideal of rational inference

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# Preface

This volume records papers given at the fourteenth international maximum entropy conference, held at St John's College Cambridge, England. It seems hard to believe that just thirteen years have passed since the first in the series, held at the University of Wyoming in 1981, and six years have passed since the meeting last took place here in Cambridge. So much has happened.

There are two major themes at these meetings, inference and physics. The inference work uses the confluence of Bayesian and maximum entropy ideas to develop and explore a wide range of scientific applications, mostly concerning data analysis in one form or another. The physics work uses maximum entropy ideas to explore the thermodynamic world of macroscopic phenomena. Of the two, physics has the deeper historical roots, and much of the inspiration behind the inference work derives from physics. Yet it is no accident that most of the papers at these meetings are on the inference side. To develop new physics, one must use one's brains alone. To develop inference, computers are used as well, so that the stunning advances in computational power render the field open to rapid advance.

Indeed, we have seen a revolution. In the larger world of statistics beyond the maximum entropy movement as such, there is now an explosion of work in Bayesian methods, as the inherent superiority of a defensible and consistent logical structure becomes increasingly apparent in practice. In principle, the revolution was overdue by some decades, as our elder statesmen such as Edwin Jaynes and Myron Tribus will doubtless attest. Yet in practice, we needed the computers: knowing what ought to be added up is of limited value until one can actually do the sums.

Here, in this series of proceedings, one can see the revolution happen as the power and range of the work expand, and the level of understanding deepens. The movement is wary of orthodoxy, and not every author (to say nothing of the editors) will agree with every word written by every other author. So, reader, scan the pages with discernment for the jewels made for you...

As a gesture of faith and goodwill, our publishers, Kluwer Academic Publishers, actively sponsored the meeting, and in this they were joined by Bruker Spectrospin and by MaxEnt Solutions Ltd. To these organisations, we express our gratitude and thanks. Our thanks also go to the staff of St John's College, Cambridge, for their efficiency and help in letting the meeting be worthy of its surroundings. Let us all go forward together.

John Skilling, Sibusiso Sibisi Cavendish Laboratory Cambridge 1995