

Nonlinear Solid Mechanics

SOLID MECHANICS AND ITS APPLICATIONS

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Adnan Ibrahimbegovic

Nonlinear Solid Mechanics

Theoretical Formulations and Finite Element
Solution Methods

 Springer

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This is a translation from the French book, *Mécanique non linéaire des solides déformables : formulation théorique et résolution numérique par éléments finis*, published by Hermes Science - Lavoisier Paris in 2006, which was nominated for the Roberval Award for University Textbooks in French.

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To my family

Foreword

It is with great pleasure that I accepted invitation of Adnan Ibrahimbegovic to write this preface, for this invitation gave me the privilege to be one of the first to read his book and allowed me to once again emphasize the importance for our discipline of solid mechanics, which is currently under considerable development, to produce the reference books suitable for students and all other researchers and engineers who wish to advance their knowledge on the subject.

The solid mechanics has closely followed the progress in computer science and is currently undergoing a true revolution where the numerical modelling and simulations are playing the central role. In the industrial environment, the ‘virtual’ (or the computing science) is present everywhere in the design and engineering procedures. I have a habit of saying that the solid mechanics has become the science of modelling and in that respect expanded beyond its traditional frontiers. Several facets of current developments have already been treated in different works published within the series ‘Studies in mechanics of materials and structures’; for example, modelling heterogeneous materials (Besson et al.), fracture mechanics (Leblond), computational strategies and namely LATIN method (Ladevèze), instability problems (NQ Son) and verification of finite element method (Ladevèze-Pelle). To these (French) books, one should also add the work of Lemaitre-Chaboche on nonlinear behavior of solid materials and of Batoz on finite element method.

The book of Adnan Ibrahimbegovic also deals with nonlinear solid mechanics, but with the unique approach of the author: each question is examined from all different facets pertaining to either mechanics, mathematics or computations with a special attention to the finite element methods. It is the main strength of this book to provide as complete as possible answer to each question. Such an exhaustive approach is also characteristic of list of different topics studied that count among them some of the main difficulties of modern mechanics: damage theory, localization and failure, discrete models, multi-physics, multi-scale, parallelism etc. Only omission are the issues of verification and validation, which are just mentioned. Quite naturally, this work

is marked by the author's research, demonstrating a thorough understanding of the modern mechanics and a number of essential personal contributions. Collecting in a single book such a broad knowledge and know-how led inevitably to restructuring the presentation of the nonlinear solid mechanics and merging successfully the European and North-American schools, for the greatest benefits of readers. Even if all points of view presented in the book will not necessarily be shared by everybody, together they provide an unusually illuminating and lively image of the subject.

Cachan, August 2006

Pierre LADEVEZE

Professor at Ecole Normale Supérieure de Cachan

EADS Foundation Chair 'Advanced Computational Structural Methods'

Professor Adnan Ibrahimbegovic belongs to a select group of active researchers and developers of the 'finite element method technology' applied to solving the nonlinear problems in mechanics of solids and structures with complex constitutive behavior under static and dynamic loading; the latter includes a wide spectrum that spans from theoretical modelling to validation tests and passing through all numerical implementation aspects. If this kind of developments have been more numerous in eighties and nineties, they have become more rare in 2006 because of the maturity that the discipline has reached as the result of research works of a very active international community of computational mechanics (which is organized within the *International Association of Computational Mechanics*), with this author as its active and established member, as well as the presence of fairly complete and efficient commercial software packages. Nevertheless, there still exist significant needs for improvement, not only in terms of theoretical formulations (variational formulations) and numerical implementation (choice of discrete models and consistent approximations), but also in terms of solution technics of discretized problems, which are nowadays highly nonlinear and non-stationary with different coupling conditions, and without forgetting the programming aspects since the final development stage is inevitably the corresponding software product.

The book 'Nonlinear solid mechanics: Theoretical formulations and finite element solution methods' reflects the rich international teaching experience to master and doctoral students, as well as the joint collaborative research with a number of renown institutions in Europe and North-America (University of California at Berkeley, Swiss Federal Institute of Technology in Lausanne, Compiègne University of Technology in France, Laval University in Quebec, Ecole Normale Supérieure in Cachan, University of Ljubljana, Technical University of Braunschweig). With this detailed and original work, Adnan Ibrahimbegovic offers to master and doctoral students, and also to

researchers and to computational software developers, the results and compilation of a number of research works.

This book deals with the analysis of deformable solids accounting for:

- Nonlinear elastic constitutive behavior, plasticity and viscoplasticity with damage, both for small and large deformations
- Contact conditions between deformable solids and the presence of inertia terms.

The theoretical and numerical aspects are pertinent to:

- Hu-Washizu variational principles and associated mixed finite element approximations and incompatible mode elements
- Time-integration schemes for inelastic constitutive models and nonlinear dynamics
- Thermomechanical coupling (also for elastoplasticity) and micro-macro approach
- Geometric and material instabilities.

We can think that the author will eventually publish one more book, dealing with structural mechanics models of rods, plates and shells, given his numerous contributions in that domain.

I wish that the present book meets the success equal to the international reputation of the author.

Compiègne, September 2006

Jean-Louis BATOZ
Professor at Compiègne University of Technology

Preface

The roots of this book go back to my doctoral studies at the University of California at Berkeley, from 1986 to 1989, funded by a Fullbright Grant. The UC Berkeley in general, and Structural Engineering, Mechanics and Materials Division in particular, provided an excellent study and research environment, with the opportunities to exchange the ideas with some extraordinary talented people from all over the world. Subsequently, I had a good fortune to stay on for another couple of years as a post-doc with my Berkeley mentors, Professor Edward L. Wilson and Professor Robert L. Taylor, which allowed me to explore a very wide variety of topics. The same good fortune was my subsequent research appointment at the Swiss Federal Institute of Technology in Lausanne at Structural and Continuum Mechanics Laboratory, directed by Professor François Frey, who granted me complete freedom to carry on with further explorations.

The work on the book started in 1994 with my first Professor appointment in France at the Compiègne University of Technology, continued from 1999 at the Ecole Normale Supérieure of Cachan in France, and kept gradually evolving as the result of a very fruitful interaction with graduate and undergraduate students, my doctoral students and colleagues of both faculties, in Cachan and in Compiègne. The final contents of the book was finally decided while preparing the IPSI course on Computational Solid and Structural Mechanics, which was taught several times in France, and more recently in Germany and in Italy, together with my colleague Robert L. Taylor, Professor at the University of California at Berkeley, to the audience of engineers coming from a number of prominent European companies, university teachers, researchers and graduate students. The first part of my sabbatical leave from ENS-Cachan in 2005 allowed me to finally converge with this long project. The work on French version of the book [115], which was published in 2006 within the collection of graduate textbooks in mechanics edited by Professor Paul Germain and Professor Pierre Ladevèze, was completed during my stay at the Swiss Federal Institute of Technology in Lausanne, as well as the subsequent stay at the University of Ljubljana, Slovenia. The work on

the present English version of the book started during the second leg of my sabbatical leave that I spent at the Technical University of Braunschweig, Germany, which was made possible by the financial support of the Alexander von Humboldt Foundation through the Research Award in Technical Mechanics for scientists with internationally recognized qualifications.

Cachan, August 2008

Adnan Ibrahimbegovic
Professor at Ecole Normale Supérieure de Cachan

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I would like to thank in particular all those who contributed to improving the book contents, either by generously supplying the results for a number of illustrative numerical simulations, or by participating in what seemed as never ending task of proofreading: some of my former doctoral students, Delphine Brancherie, Jean-Baptiste Colliat, Damijan Markovic, Norberto Dominguez, Guillaume Hervé, Lotfi Chorfi, Said Mamouri, Mazen Almikdad and Fadi Gharzeddine, and my colleagues, Catherine Knopf-Lenoir-Vayssade, Pierre Villon, François Frey, Blaise Rebora, Luc Davenne and Germaine Néfussi. Some of my present doctoral students have also contributed to this English edition, among them: Anna Kucerova, Sergiy Melnyk, Martin Hautefeuille, Amor Boulkertous, Christophe Kassiotis and Pierre Jehel. Thanks are also due to my senior colleagues, Professor Pierre Ladevèze and Professor Jean-Louis Batoz, for kindly accepting to prepare the preface for the French edition of the book, which I took the liberty to translate into English.

Last but not least, I thank my spouse Nita Ibrahimbegovic for her precious support.

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