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Geometric Q_p Functions

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Contents

Preface	ix
1 Preliminaries	1
1.1 Background	1
1.2 Logarithmic Conformal Mappings	7
1.3 Conformal Domains and Superpositions	12
1.4 Descriptions via Harmonic Majorants	16
1.5 Regularity for the Euler–Lagrange Equation	19
1.6 Notes	22
2 Poisson versus Berezin with Generalizations	25
2.1 Boundary Value and Brownian Motion	25
2.2 Derivative-free Module via Poisson Extension	29
2.3 Derivative-free Module via Berezin Transformation	32
2.4 Mixture of Derivative and Quotient	35
2.5 Dirichlet Double Integral without Derivative	40
2.6 Notes	45
3 Isomorphism, Decomposition and Discreteness	47
3.1 Carleson Measures under an Integral Operator	47
3.2 Isomorphism to a Holomorphic Morrey Space	52
3.3 Decomposition via Bergman Style Kernels	57
3.4 Discreteness by Derivatives	64
3.5 Characterization in Terms of a Conjugate Pair	67
3.6 Notes	71
4 Invariant Preduality through Hausdorff Capacity	73
4.1 Nonlinear Integrals and Maximal Operators	73
4.2 Adams Type Dualities	81
4.3 Quadratic Tent Spaces	84
4.4 Preduals under Invariant Pairing	89
4.5 Invariant Duals of Vanishing Classes	96

4.6	Notes	103
5	Cauchy Pairing with Expressions and Extremities	107
5.1	Background on Cauchy Pairing	107
5.2	Cauchy Duality by Dot Product	113
5.3	Atom-like Representations	118
5.4	Extreme Points of Unit Balls	123
5.5	Notes	133
6	As Symbols of Hankel and Volterra Operators	135
6.1	Hankel and Volterra from Small to Large Spaces	135
6.2	Carleson Embeddings for Dirichlet Spaces	138
6.3	More on Carleson Embeddings for Dirichlet Spaces	144
6.4	Hankel and Volterra on Dirichlet Spaces	150
6.5	Notes	159
7	Estimates for Growth and Decay	163
7.1	Convexity Inequalities	163
7.2	Exponential Integrabilities	169
7.3	Hadamard Convolutions	177
7.4	Characteristic Bounds of Derivatives	182
7.5	Notes	188
8	Holomorphic Q-Classes on Hyperbolic Riemann Surfaces	191
8.1	Basics about Riemann Surfaces	191
8.2	Area and Seminorm Inequalities	195
8.3	Intermediate Setting – BMOA Class	201
8.4	Sharpness	207
8.5	Limiting Case – Bloch Classes	215
8.6	Notes	223
	Bibliography	227
	Index	239

Preface

The aim of the book *Geometric Q_p Functions* is to document the rich structure of the holomorphic Q functions which are geometric in the sense that they transform naturally under conformal mappings, with particular emphasis on the last few years' development based on interaction between geometrical function and measure theory and other branches of mathematical analysis, including complex variables, harmonic analysis, potential theory, functional analysis, and operator theory.

The book comprises eight chapters in which some results appear for the first time. The first chapter begins with a motive and a very brief review of the mostly standard characterizations of holomorphic Q functions presented in the author's monograph — Springer's LNM 1767: *Holomorphic Q Classes* — followed by some further preliminaries on logarithmic conformal maps, conformal domains and superpositions and harmonic majorants with an application to Euler–Lagrange equations. The second chapter gives function-theoretic characterizations by means of Poisson extension and Berezin transform with two more generalized variants. The third chapter takes a careful look at isomorphism, decomposition, and discreteness of spaces via equivalent forms of the generalized Carleson measures. The fourth chapter discusses invariant preduality through Hausdorff capacity, which is a useful tool to classify negligible sets for various fine properties of functions. The fifth chapter develops some essential properties of the Cauchy dualities via both weak factorizations and extreme points of the target function spaces. The sixth chapter shows particularly that each holomorphic Q function can be treated as a symbol of the holomorphic Hankel and Volterra operators acting between two Dirichlet spaces. The seventh chapter deals with various size estimates involving functions and their exponentials and derivatives. Finally, the eighth chapter handles how much of the basic theory of holomorphic Q functions can be carried over the hyperbolic Riemann surfaces by sharpening the area and isoperimetric inequalities and settling the limit spaces.

Although this book may be more or less regarded as a worthy sequel to the previously-mentioned monograph, it is essentially self-contained. And so, without reading that monograph, readers can understand the contents of this successor, once they are familiar with some basic facts on geometric function-measure theory and complex harmonic-functional analysis. For further background, each chapter ends with brief notes on the history and current state of the subject. Readers may

consult those notes and go further to study the references cited by this book for more information.

As is often the case, the completion of a book is strongly influenced by some organizations and individuals. This book has been no exception. Therefore, the author would like to deliver a word of thanks to: Natural Sciences and Engineering Research Council of Canada as well as Faculty of Science, Memorial University of Newfoundland, Canada, that have made this book project possible; next, a number of people including (in alphabetical order): D. R. Adams (University of Kentucky), A. Aleman (Lund University), R. Aulaskari (University of Joensuu), H. Chen (Nanjing Normal University), K. M. Dyakonov (University of Barcelona), P. Fenton (University of Otago), T. Hempfling (Birkhäuser Verlag AG), M. Milman (Florida Atlantic University), M. Pavlovic (University of Belgrade), J. Shapiro (Michigan State University), A. Siskakis (University of Thessaloniki), K. J. Wirths (Technical University of Braunschweig), Z. Wu (University of Alabama), G. Y. Zhang (Polytechnic University of New York), R. Zhao (State University of New York at Brockport) and K. Zhu (State University of New York at Albany), who have directly or indirectly assisted in the preparation of this book. Last but not least, the author's family, who the author owes a great debt of gratitude for their understanding and moral support during the course of writing.

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