Frontiers in Mathematics

Jie Xiao

Geometric Q_p Functions

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Contents

Preface

The aim of the book *Geometric Q_p Functions* is to document the rich structure of the holomorphic Q functions which are geometric in the sense that they transform naturally under conformal mappings, with particular emphasis on the last few years' development based on interaction between geometrical function and measure theory and other branches of mathematical analysis, including complex variables, harmonic analysis, potential theory, functional analysis, and operator theory.

The book comprises eight chapters in which some results appear for the first time. The first chapter begins with a motive and a very brief review of the mostly standard characterizations of holomorphic Q functions presented in the author's monograph — Springer's LNM 1767: *Holomorphic Q Classes* — followed by some further preliminaries on logarithmic conformal maps, conformal domains and superpositions and harmonic majorants with an application to Euler–Lagrange equations. The second chapter gives function-theoretic characterizations by means of Poisson extension and Berezin transform with two more generalized variants. The third chapter takes a careful look at isomorphism, decomposition, and discreteness of spaces via equivalent forms of the generalized Carleson measures. The fourth chapter discusses invariant preduality through Hausdorff capacity, which is a useful tool to classify negligible sets for various fine properties of functions. The fifth chapter develops some essential properties of the Cauchy dualities via both weak factorizations and extreme points of the target function spaces. The sixth chapter shows particularly that each holomorphic Q function can be treated as a symbol of the holomorphic Hankel and Volterra operators acting between two Dirichlet spaces. The seventh chapter deals with various size estimates involving functions and their exponentials and derivatives. Finally, the eighth chapter handles how much of the basic theory of holomorphic Q functions can be carried over the hyperbolic Riemann surfaces by sharpening the area and isoperimetric inequalities and settling the limit spaces.

Although this book may be more or less regarded as a worthy sequel to the previously-mentioned monograph, it is essentially self-contained. And so, without reading that monograph, readers can understand the contents of this successor, once they are familiar with some basic facts on geometric function-measure theory and complex harmonic-functional analysis. For further background, each chapter ends with brief notes on the history and current state of the subject. Readers may consult those notes and go further to study the references cited by this book for more information.

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