

Electromagnetic Noise
and Quantum Optical Measurements

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Hermann A. Haus

Electromagnetic Noise and Quantum Optical Measurements

With 151 Figures and 117 Problems
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Preface

Throughout my professional career I have been fascinated by problems involving electrical noise. In this book I would like to describe aspects of electrical noise somewhat in the manner of a Russian matryoshka doll, in which each shell contains a different doll, alluding to deeper and deeper meanings hidden inside as outer appearances are peeled away.

Let us look at some dictionary definitions of noise. Surprisingly, the origin of the word in the English language is unknown. The *Oxford Universal Dictionary* (1955) has the following definition: “Noise. 1. loud outcry, clamour or shouting; din or disturbance; common talk, rumour, evil report, scandal – 1734. A loud or harsh sound of any kind; a din . . . An agreeable or melodious sound. Now rare, ME. A company or band of musicians.”

This is not a helpful definition of the technical meaning of noise. The Supplement to the *Oxford English Dictionary* (1989) lists the following: “Noise. 7. In scientific use, a collective term (used without the indefinite article) for: fluctuations or disturbances (usu. irregular) which are not part of a wanted signal, or which interfere with its intelligibility or usefulness.”

The last definition is an appropriate one and relates to the work of Prof. Norbert Wiener who developed the mathematics of statistical functions in the 1930s and 1940s. To this day I am awed by the power of mathematical prediction of averages of outcomes of statistically fluctuating quantities. These predictions extend to the theory of and experiments on noise.

Let us look at the interpretation in other languages of the word used for the technical term “noise”.

In German	<i>Rauschen</i> : rush, rustle, murmur, roar, thunder, (poet.) sough.
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In Russian	<i>shum</i> : noise, hubbub, uproar; <i>vetra, voln</i> : sound of wind, waves.
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In French	<i>bruit</i> : noise, din, racket, uproar, commotion, clamor; (fig.) tumult, sedition; fame, renown, reputation; <i>beau- coup de bruit pour rien</i> , much ado about nothing.
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In Italian	<i>rumore</i> : noise, din, clamor, outcry, uproar; rumor.
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It is interesting how different languages attach different meanings to noise. The German and Russian origins are onomatopoeic, simulating the sound of

rushing water or rustling of leaves, and do not necessarily possess the connotation of unpleasantness. The French and Italian words have more abstract meanings. Surprisingly, in French, it describes characteristics of persons who stick out, are famous. In Italian it is clearly related to the word “rumor”. The etymology of the word “noise” is a glimpse of the complexity and subtlety of the meanings attached to words by different cultures. In the world of physics and technology, noise is equally multifaceted.

A fascinating fact is that the ear is adjusted to have the highest allowed sensitivity without being disturbed by one of the fundamental sources of noise, thermal noise. Thermal noise is the agitation experienced by the molecules in gases, liquids, and solids at all temperatures above absolute zero (on the Kelvin scale). The molecules of air bounce around and hit the eardrums in a continuous pelting “rain” of particles. If the ear were sensitive to that bombardment, one would hear a continuous hissing noise comparable to that of the noise of a radio tuned between stations with the volume turned up. A simple computation finds that the power impinging upon the ear from this thermal noise is of the order of 0.3×10^{-12} W, a third of the threshold of hearing [1], a rather remarkable fact.

Many of us have experienced the strange sensation that is produced when a large shell is held to the ear. Popularly this is known as “hearing the ocean”. In fact, this effect is due to the noise of the air particles impinging upon the ear, enhanced by the shell acting as a resonator. Thus, even a normal ear can hear the air particles impinging upon the ear when the effect is enhanced by some means. Later in this book we shall learn how resonators enhance the spectrum of noise near their resonance frequency.

My interest in noise, reflected in the content of this book, was and is mainly in electrical and optical noise. It is not hard to understand the origin of electrical noise, at least the one related to the agitation of particles. Particles with charge are surrounded by fields which, in turn, produce charge accumulation (of opposite sign) in surrounding electrodes. As the particles bounce around when driven by thermal effects or quantum effects, the charges in the electrodes are dragged along and produce spurious currents, noise currents.

Electrical communications engineers worry about noise because they have to discern signals in the presence of such background noise. In all cases in which the background noise is worrisome, the signals are weak so that amplifiers are needed to raise their power to detectable levels. Amplifiers add noise of their own to the background noise. *The ultimate source of low-frequency (including microwave) amplifier noise is the “graininess of the electrical charge”*. This fact was recognized in its full significance by Schottky in his classic paper in 1918 [2]. I quote from Schottky (my English translation):

Cascading of vacuum tube amplifiers has made possible in recent years the detection and measurement of alternating currents of exceedingly small amplitude. Many technical tasks have thereby realized a sudden benefit, but also a new field of research has been opened up. The new amplifying circuits have the same impact on electrical studies as the microscope has had for optics. Because no clear limit has appeared to date on the achievable amplification, one could hope to advance to the infinitesimally small by proper shielding, interference-free layouts, etc. of the amplifying circuits; the dream of “hearing the grass grow” has appeared achievable to mankind.

This is an allusion by Schottky to the sensory power ascribed by the brothers Grimm fairy tales to particularly endowed individuals. In the sequel he shows that the dream will not come true and I quote:

The first insurmountable obstacle is provided, remarkably, by the size of the elementary quantum of electricity (the charge of the electron).

Schottky wrote his paper a decade before the formulation of the uncertainty principle of Heisenberg. Some of the noise generated in amplifiers and recognized by Schottky can be controlled. The amplifiers can be cooled or refrigerated. The shot noise can be reduced by utilizing the mutual repulsion among the negatively charged electrons. Schottky was careful to point out in his paper that, with the current densities achievable in his day, such repulsion could be ignored. In the intervening 75 years a great deal has happened and this research led to the development of ultra-low-noise amplifiers.

The fundamental limit of the noise performance of amplifiers is ultimately determined by quantum mechanics. This was the reason why I studied optical amplification, at frequencies at which the quantum effects of the electromagnetic field are observable, and at which quantum effects are, fundamentally, responsible for the noise performance of optical amplifiers. This very property of optical amplifiers makes them ideal models of quantum measurement apparatus and permits study of the theory of quantum measurement with the aid of simple optical measurement devices. This book thus spans the range from microwave propagation and amplification to optical propagation and amplification, all the way to issues of the theory of quantum measurement.

A book based on the work of 45 years clearly rests on collaboration with many individuals. Among those I should mention with gratitude are the late Prof. Richard B. Adler, Charles Freed, Dr. James Mullen, Prof. Y. Yamamoto, Dr. J. P. Gordon, and many past and present students. Among these, credit goes to Patrick Chou, John Fini, Leaf Jiang, Thomas Murphy, Steve Patterson, Michael Watts, William Wong, and Charles Yu for the careful reading of the manuscript that led to many corrections and suggestions for improvements.

VIII Preface

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Hermann A. Haus

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