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Mathematical Modeling in Epidemiology

Springer-Verlag
Berlin Heidelberg New York 1980

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ISBN-13: 978-3-540-10328-8 e-ISBN-13: 978-3-642-67795-3
DOI: 10.1007/978-3-642-67795-3

Library of Congress Cataloging in Publication Data. Frauenthal, J. C. 1944-.
Mathematical modeling in epidemiology. (Universitext). Bibliography: p.
Includes index. 1. Epidemiology-Mathematical models. 2. Mathematical models.
I. Title.
RA652.2.M3F72. 614.4'0724. 80-21231

2141/3140-543210

Preface

The text of this book is derived from courses taught by the author in the Department of Applied Mathematics and Statistics at the State University of New York at Stony Brook. The audience for these courses was composed almost entirely of fourth year undergraduate students majoring in the mathematical sciences. The students had ordinarily completed four semesters of calculus and one of probability. Few had any prior experience with differential equations, stochastic processes, or epidemiology. It also seems prudent to mention that the author's background is in engineering and applied mathematics and not in epidemiology; it is hoped that this is not painfully obvious.

The topics covered in this book have in some cases been modified from the way they were originally presented. However, care has been taken to include a suitable amount of material for a one semester course; the temptation to add gratuitous subject matter has been resisted. Similarly, when a choice between clarity and rigor was available, the more easily understood exposition was selected.

By looking only at the table of contents, the casual reader could be easily misled into thinking that the main concern of this book is with epidemiology. This is not the case. The purpose of this book is to illustrate the process of formulating and solving mathematical models. Epidemiology is employed as a pedagogic device to provide unity and intuitive appeal to the various mathematical ideas discussed; when the epidemiological terminology is stripped away, what remains is a collection of deterministic and stochastic mathematical models.

The topics ciscussed in this book fall quite naturally into two groups. The first contains general models for the spread of a disease (or rumor or altered state) through a susceptible population. Different ways of keeping track of the state of the population are considered by using different treatments of time and numbers. In Chapter 1 (Deterministic Epidemic Models), three deterministic mathematical models for an epidemic outbreak of a contagious disease are developed. Each successive model attempts to rectify the faults of the prior formulations. The final model is interesting in that it leads to the conclusion that there is a disease threshold or minimum number of susceptibles needed for the occurrence of an epidemic, a condition not intentionally built in to the mathematics. The second chapter (Rumors and Mousetraps) attempts to illustrate the distinction between a deterministic and a stochastic formulation of two epidemic-like processes. The models are first posed in deterministic terms; in each case it is soon apparent that more than one possible outcome can occur. To avoid the necessity of introducing probability densities, the expected evolution of the system is calculated. In Chapter 3 (Stochastic Epidemic Models), the models of Chapter 1 are re-derived in full stochastic form. It is discovered that the probabilistic versions are considerably harder to solve than their deterministic analogues, and that they lead to somewhat different results. Chapter 4 (Chain Binomial Models) investigates both the deterministic and stochastic versions of a model for an epidemic outbreak within a small population. The model is strongly reminiscent of the ones discussed in Chapter 2. However, the stochastic model employed conditions the present state of the system on prior states. Several methods are discussed for deconditioning the probabilities, and the outcome of the stochastic and deterministic versions are compared. Chapter 5 (Branching Process Model) investigates the application of the well-known Galton-Watson branching process to a small epidemic within a large population of susceptibles.

The second portion of the book is concerned with models for specific diseases. These have been included to allow a careful discussion of modeling only the salient points of a larger problem. Chapter 6 (Smallpox Vaccination Discontinuation) studies the question: given a rare disease for which there is a vaccine, when does it become better to risk the consequences of an epidemic than to incur the mortality associated with the vaccine? The optimal solution is found by investigating various stages of an outbreak using stochastic models. In Chapter 7 (Schistosomiasis Eradication) a mathematical model is constructed to explain a qualitative observation about schistosomiasis, a host-vector disease in which humans are the hosts and snails are the vectors. By identifying the critical phase in the disease process (which occurs in the humans' circulatory systems), a specialized model turns out to be sufficient to explain the paradox. Chapter 8 (Gonorrhea) looks at another troublesome situation: the recent rise in the number of cases of gonorrhea within the United States. The model is used to show that we are not in fact experiencing the beginning of an epidemic. The final chapter (Sickle Cell Anemia) looks at an explanation for the occurrence of a well-known genetic disease by means of models from Mendelian genetics. It is hoped that students will develop enthusiasm for mathematical applications based upon seeing situations which they understand treated with fairly sophisticated techniques.

At the end of each chapter a number of problems have been included to provide practice with mathematical concepts and techniques. Solutions will be found at the end of the book. It is suggested that during a one semester course all problems be solved by students. Without this level of activity the mathematical details cannot be adequately appreciated. Also at the end of each chapter is an annotated list of references.

Most of the ideas and models contained in this book are not original, and to their creators I owe a debt of gratitude. I would like to express my thanks in particular to Norman T.J. Bailey, Chief of the Health Statistical Methodology Section of the World Health Organization and more than anyone else the guiding force in mathematical epidemiology. I should also like to express my sincere appreciation to the Alfred P. Sloan Foundation, whose support greatly facilitated the preparation of this book.

Stony Brook, New York
April 1980

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