Classics in Mathematics

David Gilbarg • Neil S. Trudinger

Elliptic Partial Differential Equations of Second Order

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David Gilbarg was born in New York in 1918, and was educated there through undergraduate school. He received his Ph.D. degree at Indiana University in 1941. His work in fluid dynamics during the war years motivated much of his later research on flows with free boundaries. He was on the Mathematics Faculty at Indiana University from 1946 to 1957 and at Stanford University from 1957 on. His principal interests and contributions have been in mathematical fluid dynamics and the theory of elliptic partial differential equations.

Neil S. Trudinger was born in Ballarat, Australia in 1942. After schooling and undergraduate education in Australia, he completed his Ph.D. at Stanford University, USA in 1966. He has been a Professor of Mathematics at the Australian National University, Canberra since 1973. His research contributions, while largely focused on non-linear elliptic partial differential equations, have also spread into geometry, functional analysis and computational mathematics. Among honours received are Fellowships of the Australian Academy of Science and of the Royal Society of London.

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Preface to the Revised Third Printing

This revision of the 1983 second edition of "Elliptic Partial Differential Equations of Second Order" corresponds to the Russian edition, published in 1989, in which we essentially updated the previous version to 1984. The additional text relates to the boundary Hölder derivative estimates of Nikolai Krylov, which provided a fundamental component of the further development of the classical theory of elliptic (and parabolic), fully nonlinear equations in higher dimensions. In our presentation we adapted a simplification of Krylov's approach due to Luis Caffarelli.

The theory of nonlinear second order elliptic equations has continued to flourish during the last fifteen years and, in a brief epilogue to this volume, we signal some of the major advances. Although a proper treatment would necessitate at least another monograph, it is our hope that this book, most of whose text is now more than twenty years old, can continue to serve as background for these and future developments.

Since our first edition we have become indebted to numerous colleagues, all over the globe. It was particularly pleasant in recent years to make and renew friendships with our Russian colleagues, Olga Ladyzhenskaya, Nina Ural'tseva, Nina Ivochkina, Nikolai Krylov and Mikhail Safonov, who have contributed so much to this area. Sadly, we mourn the passing away in 1996 of Ennico De Giorgi, whose brilliant discovery forty years ago opened the door to the higher-dimensional nonlinear theory.

October 1997 *David Gilbarg · Neil S. Trudinger*

Preface to the First Edition

This volume is intended as an essentially self-contained exposition of portions of the theory of second order quasilinear elliptic partial differential equations, with emphasis on the Dirichlet problem in bounded domains. It grew out of lecture notes for graduate courses by the authors at Stanford University, the final material extending well beyond the scope of these courses. By including preparatory chapters on topics such as potential theory and functional analysis, we have attempted to make the work accessible to a broad spectrum of readers. Above all, we hope the readers of this book will gain an appreciation of the multitude of ingenious barehanded techniques that have been developed in the study of elliptic equations and have become part of the repertoire of analysis.

Many individuals have assisted us during the evolution of this work over the past several years. In particular, we are grateful for the valuable discussions with L. M. Simon and his contributions in Sections 15.4 to 15.8; for the helpful comments and corrections of J. M. Cross, A. S. Geue, J. Nash, P. Trudinger and B. Turkington; for the contributions of G. Williams in Section 10.5 and of A. S. Geue in Section 10.6; and for the impeccably typed manuscript which resulted from the dedicated efforts of Isolde Field at Stanford and Anna Zalucki at Canberra. The research of the authors connected with this volume was supported in part by the National Science Foundation.

August 1977

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Note: The Second Edition includes a new, additional Chapter 9. Consequently Chapters 10 and 15 referred to above have become Chapters 11 and 16.

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