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Linear Optimization and Extensions

Problems and Solutions

With 67 Figures



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Preface

Books on a technical topic – like linear programming – *without* exercises ignore the principal beneficiary of the endeavor of writing a book, namely the student – who learns best by doing exercises, of course. Books *with* exercises – if they are challenging or at least to some extent so – need a solutions manual so that students can have recourse to it when they need it. Here we give solutions to all exercises and case studies of M. Padberg's *Linear Optimization and Extensions* (second edition, Springer-Verlag, Berlin, 1999). In addition we have included several new exercises and taken the opportunity to correct and change some of the exercises of the book. Here and in the main text of the present volume the terms “book”, “text” etc. designate the second edition of Padberg's LP book and the page and formula references refer to that edition as well. All new and changed exercises are marked by a star * in this volume. The changes that we have made in the original exercises are inconsequential for the main part of the original text where several of the exercises (especially in Chapter 9) are used on several occasions in the proof arguments. None of the exercises that are used in the estimations, etc. have been changed. Quite a few exercises instruct the students to write a program in a computer language of their own choice. We have chosen to do that in most cases in MATLAB without *any* regard to efficiency, etc. Our prime goal here is to use a macro-language that resembles as closely as possible the mathematical statement of the respective algorithms. Once students master this first level, they can then go ahead and discover the pleasures and challenges of writing efficient computer code on their own.

To make the present volume as self-contained as possible, we have provided here summaries of each chapter of Padberg's LP book. While there is some overlap with the text, we think that this is tolerable. The summaries are –in almost all cases– without proofs, thus they provide a “mini-version” of the material treated in the text. Indeed, we think that having such summaries without the sometimes burdensome proofs is an advantage to the reader who wants to acquaint herself/himself with the material treated at length in the text. To make the cross-referencing with the text easy for the reader, we have numbered all chapters (and most sections and subsections) as well as the formulas in these summaries exactly like in the text. Moreover, we have reproduced here most of the illustrations of the text as we find these visual aids very helpful in communicating the material. Finally, we have reproduced here the appendices of the text as the descriptions of the cases contained therein would have taken too much space anyway.

We have worked on the production of this volume over several years and did so quite frequently at the Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB) in Berlin, Germany, where Alevras was a research fellow during some of this time. We are most grateful to ZIB's vice-president, Prof. Dr. Martin Grötschel, for his hospitality and tangible support of our endeavor. Padberg's work was also supported in part through an ONR grant and he would like to thank Dr. Donald Wagner of the Office of Naval Research, Arlington, VA, for his continued support.

New York City, January, 2001

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