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Ronald W. Shephard

Cost and Production Functions

Reprint of the First Edition



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FORWARD

Ronald Shephard's book Cost and Production Functions ranks as one of the most original contributions to economic theory of all time. This remarkable book contains a full development of three seminal ideas:

1. The duality between cost and production functions. Shephard's central idea is that technologies can be determined from two alternative and completely equivalent points of view: The production function and marginal productivities of the inputs on the one hand and the cost function and the demands for the inputs conditional on output on the other. Shephard's treatment of duality is based on convex analysis and is completely modern in its orientation.

2. Shephard's lemma. Part of the duality between cost and production functions is based on the equality between derivatives of the cost function with respect to price and factor demands conditional on output. Shephard had a full appreciation of the central importance of this idea for econometric description of technologies (see, for example, pages 28 and 52). Shephard's lemma has had far reaching influence on applied econometrics over the past decade, beginning with the work of Erwin Diewert.

3. Homotheticity and homothetic separability. Shephard developed the notion of a homothetic production function and employed the idea in function and formulating the concept of homothetic separability. The critical importance of homothetic separability to duality in the theory of aggregation and index numbers was fully appreciated by Shephard.

Like many of the most profound contributions to economic theory, Augustin Cournot's Researches into the Mathematical Principles of the Theory of Wealth being another notable example, Shephard's book began its distinguished history with a period of relative obscurity. Hirofumi Uzawa played an important role in promoting Shephard's point of view and drawing attention to his work in the 1960's. However, the full flowering of Shephard's viewpoint took place during the 1970's in the work of Daniel McFadden and his associates and in the contemporaneous

work of Shephard and his own associates, both at the University of California, Berkeley.

Shephard's book has now attained the status of a true classic in the literature of economic theory. It is most appropriate that this book be reprinted in its original form, so that students can appreciate Shephard's ideas in a presentation that is still unsurpassed in lucidity and intuitive appeal.

Dale W. Jorgenson
Harvard University, Massachusetts
Summer 1981

PREFACE TO SECOND PRINTING

From a manuscript completed in 1951, the apple green booklet appeared in 1953 largely through the efforts of Oskar Morgenstern. It had a brief shelf life at Princeton University Press, with only seven copies available in 1956. After this flurry of distribution, it settled into a state of obscurity for the following decade and thereafter for the next decade the ideas of the book began to appear.

During a meeting of an International Symposium organized by the Institute for Economic Theory and Operations Research, University of Karlsruhe (under the direction of Professor W. Eichhorn), Professor Chipman suggested to me a second printing. Professor Martin Beckmann offered to provide the printing in a series of Springer Verlag for which he was an editor. Subsequently, Professor Jorgenson offered to write a Forward for the printing summarizing his speech made on the occasion of an award to me of Dr. Rer. Pol. H. C. in Economic Science at the University of Karlsruhe. Thus, this second printing came about.

I recall that Morgenstern was mystified at the complete lack of response when the book was published. I like to think that this second printing accompanied by Professor Jorgenson's remarks is a tribute to his farsighted judgement. Also that of my wife Hilda is involved. Her per-

sistence in getting me to obtain and renew the copyright
has made this printing possible.

Ronald W. Shephard
Berkeley, California
Summer 1981

PREFACE

This study is the result of an interest in the economic theory of production intermittently pursued during the past three years. Over this period I have received substantial support from the Office of Naval Research, first from a personal service consulting contract directly with the Mathematics Division of the Office of Naval Research and secondly from Project N6 onr-27009 at Princeton University under the direction of Professor Oskar Morgenstern. Grateful acknowledgement is made to the Office of Naval Research for this support and to Professor Morgenstern, in particular, for his interest in the publication of this research.

The responsibility for errors and omissions, however, rests entirely upon the author.

Professor G. C. Evans has given in terms of a simple total cost function, depending solely upon output rate, a treatment of certain aspects of the economic theory of production which has inherent generality and convenience of formulation. The classical approach of expressing the technology of production by means of a production function is potentially less restrictive than the use of a simple total cost function, but it has not been applied in a more general form other than to derive the familiar conditions between marginal productivities of the factors of production and their market prices. Various uses of either a total cost function depending only upon output rate or an equally simplified production function have been made in both theoretical and econometric studies, but the precise connection between cost and production function of a

rationally organized process has not heretofore been fully developed. The content of Sections 1 through 5 is directed to the formulation of an integrated theory of cost and production functions, and has been stimulated in a large part by the aim to extend the basis of Evans' static and dynamic economic analyses.

The recent development of linear programming models has been instrumental in clarifying the meaning of the classical production function by constructing it as a family of efficient polyhedral surfaces in the factor space from a set of alternative elementary activities involving simple proportionality of input to output. To make this construction, strictly additive activities have been used, resulting in a production function which is homogeneous of degree one in the amounts of the factors of production, a specialization which does not entirely correspond to theoretical precepts or to observed phenomena.

This attention to the microscopic components of a production process is based upon the premise that in some way or another engineering or technical data are available and can be assembled in great detail. There are processes, e.g. some extractions and refinement of basic materials, for which this premise may be appropriate, but the large bulk of manufacturing processes are not simple enough to be analyzed a priori in this great detail, a fact which is verified, I believe, by the experience of industrial production control.

In the absence of these foreknown technological data, some consistent statistical estimating procedure must be used to derive the technological input-output coefficients for each of the possible alternative basic activities, and this estimating problem is equivalent to the formidable task of measurement of the coordinates of all vertices of the polyhedral production surface isoquant.

Since the activity analysis model of production is an approximate theoretical structure, the individual vertices of the polyhedral surface have no particular merit

in themselves, from a phenomenological point of view, and one is inclined to erase them by substitution of a surface with continuous derivatives which is represented mathematically by an expression with a few parameters to be estimated. Then we will have returned to the problem of estimating the classical production function isoquants.

There are strong reasons for expressing the viewpoint that some more aggregated form of the classical problem of measurement of a production function should be pursued, without constant returns to scale, in preference to a microscopic study like that of the activity analysis model of production. But this pursuit does not entail ignoring available technological information as to the structure of production. In fact, such knowledge should be incorporated in some convenient way into the mathematical form of the production function. Sections 6 through 9 of this study are directed to the development of a basis for carrying out this viewpoint.

Section 10 is a reformulation of Evans' dynamic analysis of monopoly in terms of a cost function which allows for anticipations by the entrepreneur of changes in cost associated with prospective variations in the prices of the factors of production.

Various portions of the material contained in the subsequent pages have been presented at the summer meetings of the Econometric Society during the years 1949 and 1950.

Ronald Shephard

October, 1951
Pacific Palisades, California

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