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Entanglement Measures and Their Properties in Quantum Field Theory

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Contents

Abstract

An entanglement measure for a bipartite quantum system is a state functional that vanishes on separable states and that does not increase under separable (local) operations. For pure states, essentially all entanglement measures are equal to the v. Neumann entropy of the reduced state, but for mixed states this uniqueness is lost. In quantum field theory, bipartite systems are associated with causally disjoint regions. But if these regions touch each other, there are no separable normal states to begin with, and one must hence leave a finite "safety corridor" between the regions. Due to this corridor, the normal states of bipartite systems are necessarily mixed, so the v. Neumann entropy is not a good entanglement measure anymore in this sense. In this volume, we study various good entanglement measures. In particular, we study the relative entanglement entropy, E_R, defined as the minimum relative entropy between the given state and an arbitrary separable state. We establish upper and lower bounds on this quantity in several situations: (1) In arbitrary CFTs in $d+1$ dimensions, we provide an upper bound for the entanglement measure of the vacuum state if the two regions of the bipartite system are a diamond and the complement of another diamond. The bound is given in terms of the spins, dimensions of the CFT, and the geometric invariants associated with the regions. (2) In integrable models in $1+1$ dimensions defined by a general analytic, crossing symmetric two-body scattering matrix, we give an upper bound for the entanglement measure of the vacuum state for a pair of diamonds that are far apart, showing exponential decay with the distance between the diamonds. The class of models includes, e.g., the Sinh–Gordon field theory. (3) We give upper bounds for our entanglement measure for a free Klein–Gordon/Dirac field in the ground state on an arbitrary static spacetime. Our upper bounds show exponential decay of the entanglement measure for large geodesic distance and an "area law" for small distances (modified by a logarithm). (4) We show that if we add charged particles to an arbitrary state, then E_R decreases by a positive amount which is no more than the logarithm of the quantum dimension of the charges (this dimension need not be an integer). (5) We establish a lower bound on our entanglement measure for arbitrary regions that get close to each other. This lower bound is of the type of an "area law" with the proportionality constant given by the number N of free fields in the UV fixed point times a quantity D_2 that can be interpreted as the distillable entanglement of one "Cbit pair" in the state.