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J.-B. Bru · W. de Siqueira Pedra

Lieb-Robinson Bounds for Multi-Commutators and Applications to Response Theory

J.-B. Bru
Departamento de Matemáticas, Facultad de
Ciencia y Tecnología
Universidad del País Vasco
Bilbao
Spain

W. de Siqueira Pedra
Department of Mathematical Physics,
Institute of Physics
University of São Paulo
São Paulo
Brazil

and

IKERBASQUE, Basque Foundation
for Science
Bilbao
Spain

and

BCAM—Basque Center for Applied
Mathematics
Bilbao
Spain

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Abstract

We generalize to multi-commutators the usual Lieb–Robinson bounds for commutators. In the spirit of constructive QFT, this is done so as to allow the use of combinatorics of minimally connected graphs (tree expansions) in order to estimate time-dependent multi-commutators for interacting fermions. Lieb–Robinson bounds for multi-commutators are effective mathematical tools to handle analytic aspects of the dynamics of quantum particles with interactions which are non-vanishing in the whole space and possibly time-dependent. To illustrate this, we prove that the bounds for multi-commutators of order three yield existence of fundamental solutions for the corresponding non-autonomous initial value problems for observables of interacting fermions on lattices. We further show how bounds for multi-commutators of an order higher than two can be used to study linear and non-linear responses of interacting fermions to external perturbations. All results also apply to quantum spin systems, with obvious modifications. However, we only explain the fermionic case in detail, in view of applications to microscopic quantum theory of electrical conduction discussed here and because this case is technically more involved.