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Dieter Britz • Jörg Strutwolf

Digital Simulation in Electrochemistry

Fourth Edition

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Dieter Britz
Aarhus University
Aarhus, Denmark

Jörg Strutwolf
Department of Chemistry
University of Tübingen
Tübingen, Germany

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This book is dedicated to our wives,
Sandra and Nooshin.

The original version of this book was revised. An erratum to this book can be found at DOI 10.1007/978-3-319-30292-8_18

Preface

This book is a revision of the 2005 3rd Edition with the same title. There is additional material, mainly in the entirely new Chap. 13 on migration and substantial new parts on ultramicroelectrode arrays in Chap. 12, as well as a general overhaul of all other chapters including many references to more recent work.

It has been our aim since the 1st Edition to write on the subject in a didactic manner, and we believe we have succeeded. The first chapters provide the reader with the background needed for the programming of simulations. There are still very few books in this field. Another book on the same subject has recently appeared [1], emphasizing cyclic voltammetry, although other experimental techniques are mentioned. The interesting book [2] by Honeychurch, very wide-ranging, presenting Mathematica [3] solutions to simulation, is now out of print and hard to obtain and in any case serves a different purpose, programming in that environment, whereas we continue to propagate a compiled language, in our case Fortran 95 (or higher).

The book is accompanied by a number of example procedures and programs, all in Fortran 90/95 (available at http://extras.springer.com). These have all been verified as far as possible. While some errors might remain, they are hopefully very few and mainly cosmetic.

We gratefully acknowledge a number of colleagues for fruitful discussions. They are (in alphabetical order) Dr. Lesław Bieniasz (Cracow University of Technology), Dr. Stephen Feldberg (Brookhaven National Laboratory, Upton, New York), Professor Bertel Kastening (Hamburg University), Professor J.A. Manzanares (Valencia University), Professor Keith Oldham (Trent University Canada), Dr. Ole Østerby (Aarhus University), and Professor Bernd Speiser (Tübingen University). If we have left anybody out, we apologize.

Finally, one of us (DB) thanks his wife for mathematical support, and we both thank our wives for their patience during the writing of this book.

Aarhus, Denmark
Tübingen, Germany
January 2016

Dieter Britz
Jörg Strutwolf

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