Probability, Stochastic Processes, and Queueing Theory

Randolph Nelson

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The Mathematics of Computer Performance Modeling

With 68 Figures

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To my Mother,

and to the memory of my Father.

For Cynthia.

Preface

Notes on the Text

We will occasionally footnote a portion of text with a "**" to indicate that this portion can be initially bypassed. The reasons for bypassing a portion of the text include: the subject is a special topic that will not be referenced later, the material can be skipped on first reading, or the level of mathematics is higher than the rest of the text. In cases where a topic is self-contained, we opt to collect the material into an appendix that can be read by students at their leisure.

Notes on Problems

The material in the text cannot be fully assimilated until one makes it "their own" by applying the material to specific problems. Self-discovery is the best teacher and although they are no substitute for an inquiring mind, problems that explore the subject from different viewpoints can often help the student to think about the material in a uniquely personal way. With this in mind, we have made problems an integral part of this work and have attempted to make them interesting as well as informative.

Many difficult problems have deceptively simple statements, and to avoid the frustration that comes from not being able to solve a seemingly easy problem, we have classified problems as to their type and level of difficulty. Any classification scheme is subjective and is prone to underestimate the difficulty of a problem (all problems are easy once one knows their solution). With this in mind, ratings should only be used as a guideline to the difficulty of problems.

The rating consists of two parts: the type of problem and its difficulty.

There are three types:

E — an exercise that requires only algebraic manipulation

M — a mathematical derivation is required to do the problem

R — the problem is a research problem.

An exercise is distinguished from a derivation in that the answer proceeds directly from material in the text and does not require additional analysis. Research problems differ from derivations in that they require more independent work by the student and may require outside reading. For each type of problem, there are three levels of difficulty:

1 — easy and straightforward,

2 — moderate difficulty,

3 — hard or requires a critical insight.

Combining these parts yields nine different ratings. For example, a rating of E2 corresponds to an exercise that requires a moderate amount of calculation, and a rating of M3 is a difficult mathematical derivation that requires some clever reasoning. Research problems have the following interpretations:

R1 — A discussion problem that is posed to provoke thought,

R2 — A problem that requires a nonobvious creative solution,

R3 — A problem that could be considered to be a research contribution.

Katonah, New York　　　　Randolph Nelson

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Finally, I would like to acknowledge my thesis advisor, Leonard Kleinrock, for the thrill of first seeing the beauty of applied probability through his eyes.

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