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Sampling With
Unequal Probabilities



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PREFACE

Work for this monograph on sampling with unequal probabilities was started when Muhammad Hanif was a visitor to the then Commonwealth Bureau of Census and Statistics, Canberra, in 1969. It remained in abeyance until he again visited Canberra, this time the Australian National University's Survey Research Centre in 1978 as Visiting Fellow. The work was substantially completed when K.R.W. Brewer visited El-Fateh University during January 1980 as Visiting Professor. Finally, in 1982 the Bibliography was revised and corrected, and a number of references added which do not appear in the text. These are indicated by an asterisk (*).

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Canberra
May 1982.

K.R.W. Brewer
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