

Variational Methods for Free Surface Interfaces

Variational Methods for Free Surface Interfaces

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Edited by
Paul Concus and Robert Finn

Organizing Committee

R. Brown, Massachusetts Institute of Technology

P. Concus, University of California, Berkeley

R. Finn (Chairman), Stanford University

S. Hildebrandt, University of Bonn

M. Miranda, University of Trento

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Paul Concus
Lawrence Berkeley Laboratory
and Department of Mathematics
University of California
Berkeley, California 94720
U.S.A.

Robert Finn
Department of Mathematics
Stanford University
Stanford, California 94305
U.S.A.

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Preface

Vallombrosa Center was host during the week September 7–12, 1985 to about 40 mathematicians, physical scientists, and engineers, who share a common interest in free surface phenomena. This volume includes a selection of contributions by participants and also a few papers by interested scientists who were unable to attend in person. Although a proceedings volume cannot recapture entirely the stimulus of personal interaction that ultimately is the best justification for such a gathering, we do offer what we hope is a representative sampling of the contributions, indicating something of the varied and interrelated ways with which these classical but largely unsettled questions are currently being attacked.

For the participants, and also for other specialists, the 23 papers that follow should help to establish and to maintain the new ideas and insights that were presented, as active working tools. Much of the material will certainly be of interest also for a broader audience, as it impinges and overlaps with varying directions of scientific development.

On behalf of the organizing committee, we thank the speakers for excellent, well-prepared lectures. Additionally, the many lively informal discussions did much to contribute to the success of the conference.

The participants benefited greatly from the warm and pleasant ambience provided by the Vallombrosa Center and its friendly and helpful staff, to whom we wish to offer our special thanks. The conference was made possible in part by support from the Air Force Office of Scientific Research, the Department of Energy, the National Science Foundation, and the Office of Naval Research. The National Science Foundation served as coordinating agency.

Paul Concus
Robert Finn

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List of Contributors

F.J. ALMGREN, JR., Mathematics Department, Princeton University, Princeton, New Jersey 08903, U.S.A.

ILYA J. BAKELMAN, Department of Mathematics, Texas A&M University, College Station, Texas 77843, U.S.A.

JOSEF BEMELMANS, Fachbereich Mathematik, Universität des Saarlandes, 6600 Saarbrücken, Federal Republic of Germany

MARIE-FRANÇOISE BIDAUT-VERON, Department of Mathematics, University of Tours, 37200 Tours, France

FREDERIC BRULOIS, California State University, Dominguez Hills, Carson, California 90747, U.S.A.

PAUL CONCUS, Lawrence Berkeley Laboratory and Department of Mathematics, University of California, Berkeley, California 94720, U.S.A.

ROBERT FINN, Department of Mathematics, Stanford University, Stanford, California 94305, U.S.A.

MICHAEL GRÜTER, University of Bonn, Mathematics Institute, 5300 Bonn, Federal Republic of Germany

S. HILDEBRANDT, University of Bonn, Mathematics Institute, 5300 Bonn, Federal Republic of Germany

DAVID A. HOFFMAN, Department of Mathematics, University of Massachusetts, Amherst, Massachusetts 01003, U.S.A.

JÜRGEN JOST, University of Bochum, Mathematics Institute, 4630 Bochum-Querenburg, Federal Republic of Germany

- NICHOLAS J. KOREVAAR, Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506, U.S.A.
- DIETMAR KRÖNER, University of Bonn, Mathematics Institute, 5300 Bonn, Federal Republic of Germany
- KIRK E. LANCASTER, Department of Mathematics and Statistics, Wichita State University, Wichita, Kansas 67208, U.S.A.
- GARY M. LIEBERMAN, Department of Mathematics, Iowa State University, Ames, Iowa 50011, U.S.A.
- MARIO MIRANDA, Institute of Mathematics, University of Trento, 38100 Trento, Italy
- AVINOAM NIR, Department of Chemical Engineering, Technion, Haifa 32000, Israel
- JAMES A. SETHIAN, Lawrence Berkeley Laboratory and Department of Mathematics, University of California, Berkeley, California 94720, U.S.A.
- DAVID SIEGEL, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1 Canada
- MICHAEL STRUWE, ETH-Zentrum, Mathematics Institute, CH-8092 Zürich, Switzerland
- LUEN-FAI TAM, Department of Mathematics, University of Illinois, Chicago, Illinois 60680, U.S.A.
- I. TAMANINI, University of Trento, Department of Mathematics, 38050 Trento, Italy
- JEAN E. TAYLOR, Mathematics Department, Rutgers University, New Brunswick, New Jersey 08903, U.S.A.
- THOMAS I. VOGEL, Department of Mathematics, Texas A&M University, College Station, Texas 77843, U.S.A.
- HENRY C. WENTE, Department of Mathematics, University of Toledo, Toledo, Ohio 43606, U.S.A.
- DANIEL ZINEMANAS, Department of Chemical Engineering, Technion, Haifa 32000, Israel