Stochastic Finite Elements:
A Spectral Approach

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To my Mother and my Father.

R.G.G.

To my parents Demetri and Aicaterine, my first mentors in quantitative thinking; to my wife Olympia, my permanent catalyst in substantive living; and to my children Demetri and Evie, a perpetual source of delightful randomness.

P.D.S.

" ... The principal means for ascertaining truth - induction and analogy - are based on probabilities; so that the entire system of human knowledge is connected with the theory (of probability) ... "

Pierre Simon de Laplace,
A Philosophical Essay on Probability, 1816.

" ... Nature permits us to calculate only probabilities, yet science has not collapsed."

Richard P. Feynman,
QED: The Strange Theory of Light and Matter, 1985.

Preface

This monograph considers engineering systems with random parameters. Its context, format, and timing are correlated with the intention of accelerating the evolution of the challenging field of Stochastic Finite Elements. The random system parameters are modeled as second order stochastic processes defined by their mean and covariance functions. Relying on the spectral properties of the covariance function, the Karhunen-Loeve expansion is used to represent these processes in terms of a countable set of uncorrelated random variables. Thus, the problem is cast in a finite dimensional setting. Then, various spectral approximations for the stochastic response of the system are obtained based on different criteria. Implementing the concept of Generalized Inverse as defined by the Neumann Expansion, leads to an explicit expression for the response process as a multivariate polynomial functional of a set of uncorrelated random variables. Alternatively, the solution process is treated as an element in the Hilbert space of random functions, in which a spectral representation in terms of the Polynomial Chaoses is identified. In this context, the solution process is approximated by its projection onto a finite subspace spanned by these polynomials.

The concepts presented in this monograph can be construed as extensions of the spectral formulation of the deterministic finite element method to the space of random functions. These concepts are further elucidated by applying them to problems from the field of structural mechanics. The corresponding results are found in agreement with those obtained by a Monte-Carlo simulation solution of the problems.

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R.G. Ghanem
P.D. Spanos
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Contents

PREFACE vii

1 INTRODUCTION 1
1.1 Motivation 1
1.2 Review of Available Techniques. 3
1.3 The Mathematical Model 9
1.4 Outline 12

2 REPRESENTATION OF STOCHASTIC PROCESSES 15
2.1 Preliminary Remarks 15
2.2 Review of the Theory 16
2.3 Karhunen-Loeve Expansion 20
2.3.1 Derivation 20
2.3.2 Properties 24
2.3.3 Solution of the Integral Equation 27
2.4 Homogeneous Chaos 45
2.4.1 Preliminary Remarks 45
2.4.2 Definitions and Properties 47
2.4.3 Construction of the Polynomial Chaos 50

3 STOCHASTIC FINITE ELEMENT METHOD: Response Representation 67
3.1 Preliminary Remarks 67
3.2 Deterministic Finite Elements 69
3.2.1 Problem Definition 69
3.2.2 Variational Approach 70
3.2.3 Galerkin Approach 72
3.2.4 p-Adaptive Methods, Spectral Methods and Hierarchical Finite Element Bases 73

3.3 Stochastic Finite Elements 74
3.3.1 Preliminary Remarks 74
3.3.2 Monte Carlo Simulation (MCS) 75
3.3.3 Perturbation Method 77
3.3.4 Neumann Expansion Method 79
3.3.5 Improved Neumann Expansion 81
3.3.6 Projection on the Homogeneous Chaos 85
3.3.7 Geometrical and Variational Extensions 97

4 STOCHASTIC FINITE ELEMENTS: Response Statistics 101
4.1 Reliability Theory Background 101
4.2 Statistical Moments 104
4.2.1 Moments and Cummulants Equations 104
4.2.2 Second Order Statistics 113
4.3 Approximation to the Probability Distribution 115
4.4 Reliability Index and Response Surface Simulation . . 118

5 NUMERICAL EXAMPLES 121
5.1 Preliminary Remarks 121
5.2 One Dimensional Static Problem 122
5.2.1 Formulation 122
5.2.2 Results 127
5.3 Two Dimensional Static Problem 150
5.3.1 Formulation 150
5.3.2 Results 160
5.4 One Dimensional Dynamic Problem 177
5.4.1 Description of the Problem 177
5.4.2 Implementation 178
5.4.3 Results 181

6 SUMMARY AND CONCLUDING REMARKS 185

BIBLIOGRAPHY 193

INDEX 211