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Smoothness Priors Analysis of Time Series



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Preface

The work described here is an attempt to integrate into a cohesive form, the papers that we have written and some previously unpublished work on the subject of “smoothness priors” and in particular, on the state space smoothness priors modeling of time series.

Originally smoothness priors as done in Shiller (1973) and Akaike (1980), was a linear model Gaussian stochastic regression least squares computational treatment of scalar and bivariate stationary and scalar nonstationary mean time series. Beginning in 1981, our own work independently and together, continued on the Shiller-Akaike path and also introduced linear Gaussian state space smoothness priors modeling of time series that were not considered by Shiller or Akaike.

The smoothness priors method is essentially quasi-Bayesian. A prior distribution on the model coefficients is parameterized by hyperparameters which in turn have a crucial role in the analysis. The maximization of the likelihood of a small number of hyperparameters permits the robust modeling of a time series with relatively complex structure and a very large number of implicitly inferred parameters. The critical statistical ideas in smoothness priors are the likelihood of the Bayesian model and the use of likelihood as a measure of the goodness of fit of the model. The Bayesianess provides a framework for doing statistical inference.

The state space modeling of not necessarily linear, not necessarily Gaussian time series is due to Genshiro Kitagawa, beginning with Kitagawa (1987). That work evolved in several stages. Kitagawa (1987) achieved general state space modeling in which the recursive conditional distributions for prediction, filtering and smoothing were successfully realized using numerical integration. That methodology was satisfactory for the modeling of non stationary mean data with abrupt discontinuities, nonstationary variance, nonstationary covariance data, nonhomogeneous discrete time series data, and non-linear time series data. Each of those models was realized with a relatively small number of states. The modeling of linear not necessarily Gaussian state space problems, with a larger number of states motivated a Gaussian mixture distribution-two filter smoothing formula to approximate the conditional densities involved. The most recent development is a general state space, (hence suitable for nonlinear modeling), Monte Carlo “particle-path tracing” method in which the distributions are approximated by many realizations. The Monte Carlo method is applicable to non Gaussian, nonstationary, nonlinear state space modeling for both small and large numbers of states.

Akaike (1968) and Pagano (1978) introduced an alternative instantaneous response orthogonal innovations representation of the multivariate stationary autoregressive time series model. We exploited that representation in a “one channel at-a-time”

paradigm to realize the modeling of multivariate nonstationary covariance time series.

In addition to presenting the various methods of state space modeling, a substantial number of applications of the modeling methodologies including, seasonal time series, discrete time processes, quasi-periodic processes, nonlinear state estimation and smoothing, the modeling of a large data set with missing data and outliers and a hidden Markov state classification procedure are shown. We expect that some of the data and programs will be distributed through S-news.

We are indebted to Professor Hirotugu Akaike, former Director General of the Institute of Statistical Mathematics, Tokyo, Japan first for the intellectual stimulation which motivated our own work in smoothness priors, and for his continuing encouragement in our efforts and also for encouraging us to work together in Tokyo to complete this manuscript. We are also indebted to Dr. David Findley, who suggested that we work together in Washington D.C. at the Division of Statistical Research of the Census Bureau of the U.S. Department of Commerce in 1981-1982. We were supported in that work by an American Statistical Association grant.

We also appreciate and thank the coauthors of several of our papers. Takanami, Jiang and Matsumoto contributed to some of the Kitagawa papers and Stone contributed to work with Gersch. We are both very appreciative of the tireless efforts of Mrs. Ono for her help in preparing the manuscript. Also we are especially grateful to and indebted to Professor David Brillinger, University of California, Berkeley, whose careful editing of our manuscript and his very constructive suggestions have helped improve the manuscript. Finally, Will Gersch is grateful to the Ministry of Education, Science, Culture and Sports of Japan for providing financial support to work in Japan.

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