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Edwige Godlewski • Pierre-Arnaud Raviart

Numerical Approximation of Hyperbolic Systems of Conservation Laws

Second Edition

Springer

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Preface to the Second Edition

There was an obvious need to complete the first edition of this textbook with the treatment of source terms. Thus, a new chapter (Chap. VII) has been added, which also provides a few important principles concerning non-conservative systems that are naturally introduced with the derivation of well-balanced or asymptotic preserving schemes. Note that most theoretical results are only referred to since it is out of scope to give detailed proofs; these may be tricky and are often quite technical.

We took the opportunity of this second edition to include more examples in the introduction chapter (now Chap. I), such as MHD, shallow water, and flow in a nozzle, and to give some insights on multiphase flow models; this last subject deserves a much longer treatment. Then we thought it is important to emphasize the change of frame from Eulerian to Lagrangian coordinates and the specificity of fluid systems. Additionally, the low Mach limit has been addressed in the chapter devoted to multidimensional systems (now Chap. V) with the final section introducing all Mach schemes.

For 25 years, there has been a tremendous lot of work dedicated to the numerical approximation of hyperbolic systems, among which we choose to introduce the relaxation approach, now at the end of Chap. IV and the case of discontinuous fluxes, and interface coupling, a topic covered in Chap. VII. Both subjects are treated in some specific outlines.

Then, some complements may be found here and there, such as recalling some results of our earlier publication at the beginning of Chap. IV, or more examples of systems of two equations in Chap. II.

We must finally confess that it took us some time to complete the work of this second edition, for different reasons. In fact, most of this work was achieved several years ago, which may explain why only few very recent results are presented, some of them are just mentioned in the notes at the end of each chapter, to give a hint and provide references where the subject is more thoroughly treated.

Preface to the First Edition

This work is devoted to the theory and approximation of nonlinear hyperbolic systems of conservation laws in one or two space variables. It follows directly a previous publication on hyperbolic systems of conservation laws by the same authors, and we shall make frequent references to Godlewski and Raviart (1991) (hereafter noted G.R.), though the present volume can be read independently. This earlier publication, apart from a first chapter, especially covered the scalar case. Thus, we shall detail here neither the mathematical theory of multidimensional *scalar* conservation laws nor their approximation in the one-dimensional case by finite-difference conservative schemes, both of which were treated in G.R., but we shall mostly consider systems. The theory for systems is in fact much more difficult and not at all completed. This explains why we shall mainly concentrate on some theoretical aspects that are needed in the applications, such as the solution of the Riemann problem, with occasional insights into more sophisticated problems.

The present book is divided into six chapters, including an introductory chapter[1]. For the reader's convenience, we shall resume in this Introduction the notions that are necessary for a self-sufficient understanding of this book –the main definitions of hyperbolicity, weak solutions, and entropy– present the practical examples that will be thoroughly developed in the following chapters, and recall the main results concerning the scalar case.

Chapter I is devoted to the resolution of the Riemann problem for a general hyperbolic system in one space dimension, introducing the classical notions of Riemann invariants and simple waves, the rarefaction and shock curves, and characteristics and entropy conditions. The theory is then applied to the p-system.

In Chap. II, we make a closer study of the one-dimensional system of gas dynamics. We solve the Riemann problem in detail and then present the

[1] The numbering of the chapters has changed in the second edition, the Introduction is now Chap. I. Hence in what follows, Chap. I refers to what is now Chap. II and so on.

simplest models of reacting flow, first the Chapman-Jouguet theory and then the Z.N.D. model for detonation.

After this theoretical approach, we go into the numerical approximation of hyperbolic systems by conservative finite-difference methods. The most usual schemes for one-dimensional systems are developed in Chap. III, with special emphasis on the application to gas dynamics. The last section begins with a short account on the kinetic theory so as to introduce kinetic schemes.

Chapter IV is devoted to the study of finite volume methods for bidimensional systems, preceded by some theoretical considerations on multidimensional systems.

For the sake of completeness, we could not avoid the problem of boundary conditions. Chapter V is but an introduction to the complex theory and presents some numerical boundary treatment.

The authors wish to thank R. Abgrall, F. Coquel, F. Dubois, and particularly T. Gallouet, B. Perthame, and D. Serre, from whom they learned a great deal and who answered willingly and most amiably their many questions.

They owe thanks to the SMAI reading committee and to the reviewers, who made very valuable suggestions.

The first author is grateful to all her colleagues who encouraged her in completing this huge work, especially to H. Le Dret and F. Murat for so often giving her their time, and to L. Ruprecht for her kind and competent assistance in the retyping of the final manuscript; such friendly help was invaluable.

Paris, France E. Godlewski and P.-A. Raviart
September 1995

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