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Continuous-Time Markov Chains and Applications

A Singular Perturbation Approach

Springer

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To Our Mentors

Harold J. Kushner and Wendell H. Fleming

Contents

Preface

This book is concerned with continuous-time Markov chains. It develops an integrated approach to singularly perturbed Markovian systems, and reveals interrelations of stochastic processes and singular perturbations. In recent years, Markovian formulations have been used routinely for numerous real-world systems under uncertainties. Quite often, the underlying Markov chain is subject to rather frequent fluctuations and the corresponding states are naturally divisible to a number of groups such that the chain fluctuates very rapidly among different states within a group, but jumps less frequently from one group to another. Various applications in engineering, economics, and biological and physical sciences have posed increasing demands on an in-depth study of such systems. A basic issue common to many different fields is the understanding of the distribution and the structure of the underlying uncertainty. Such needs become even more pressing when we deal with complex and/or large-scale Markovian models, whose closed-form solutions are usually very difficult to obtain.

Markov chain, a well-known subject, has been studied by a host of researchers for many years. While nonstationary cases have been treated in the literature, much emphasis has been on stationary Markov chains and their basic properties such as ergodicity, recurrence, and stability. In contrast, this book focuses on singularly perturbed nonstationary Markov chains and their asymptotic properties.

Singular perturbation theory has a long history and is a powerful tool for a wide variety of applications. Complementing to the ever growing literature in singular perturbations, by using the basic properties of Markov chains, this book aims to provide a systematic treatment for singularly perturbed Markovian models. It collects a number of ideas on Markov chains and singular perturbations scattered through the literature.

This book reports our recent research findings on singularly perturbed Markov chains. We obtain asymptotic expansions of the probability distributions, validate the asymptotic series, deduce the error estimates, establish asymptotic normality, derive exponential type of bounds, and investigate the structure of the weak and strong interactions. To demonstrate the applicability of the asymptotic theory, we focus on hierarchical production planning of manufacturing systems, Markov decision processes, and control and optimization of stochastic dynamic systems. Since numerical methods are viable and indispensable alternatives to many applications, we also consider numerical solutions of control and optimization problems involving Markov chains and provide computationally feasible algorithms.

Originating from a diverse range of applications in production planning, queueing network, communication theory, system reliability, and control and optimization of uncertain systems, this book is application oriented. It is written for applied mathematicians, operations researchers, physical scientists, and engineers. Selected material from the book can also be used for a one semester course for advanced graduate students in applied probability and stochastic processes.

We take great pleasure to acknowledge those who have made it possible for us to bring the book into being. We express our profound gratitude to Wendell Fleming and Harold Kushner, who introduced the intellectual horizon–stochastics to us and whose mathematical inspiration and constant encouragement have facilitated our progress. We have had the privilege to work with Rafail Khasminskii and have greatly benefited from his expertise in probability and singular perturbations. We are very much indebted to Alain Bensoussan, Wendell Fleming, Ruihua Liu, Zigang Pan, Zeev Schuss and the four reviewers for their reviews of earlier versions of the manuscript, and for their comments, criticisms and suggestions. Our thanks also go to Petar Kokotovic for providing us with references that led to further study, investigation, and discovery. We have benefited from discussions with Thomas Kurtz, whose suggestions are very much appreciated. We are very grateful to the series editor Ioannis Karatzas for his encouragement, and to the Springer-Verlag senior editor of statistics John Kimmel and the Springer-Verlag professionals for their help in finalizing the book. This research has been supported in part by the National Science Foundation and the Office of Naval Research, to whom we extend our hearty thanks.

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Notation

This section clarifies the numbering system and cross-reference conventions used in the book, and gives a glossary of symbols frequently used in the subsequent chapters. Within a chapter, equations are numbered consecutively, e.g., (3.10) indicates the tenth equation in Chapter 3. Corollaries, definitions, examples, lemmas, propositions, remarks, and theorems are treated as one entity and numbered sequentially throughout the chapter, e.g., Definition 4.1, Theorem 4.2, Corollary 4.3, etc. Likewise, assumptions are also marked consecutively within a chapter, e.g., (A6.1) stands for Assumption 1 of Chapter 6. For cross reference either within the chapter or to another chapter, equations are identified by the chapter number and the equation number, e.g., (5.2) refers to Equation 2 of Chapter 5. Similar methods apply to theorems, remarks, assumptions, etc.

Throughout the book, all deterministic processes are assumed to be Borel measurable and all stochastic processes are assumed to be measurable with respect to a given filtration. The notation $|\cdot|$ denotes either an Euclidean norm or a norm on the appropriate function spaces, which will be clear from the context. The ith component of a vector $z \in \mathbb{R}^r$ is denoted by z_i. A superscript with a pair of parentheses denotes a sequence, for example, $x^{(n)}$, $y^{(k)}$, etc. To help the reading, we provide a glossary of the symbols frequently used in what follows.

A'	transpose of a matrix (or a vector) A
B^c	complement of a set B
$\mathrm{Cov}(\zeta)$	covariance of a random variable ζ

$C^r[0,T]$	space of continuous $\mathbb{R}^r$-valued functions
C_L^2	space of functions with bounded derivatives up to the second order and Lipschitz second derivatives
$D^r[0,T]$	space of right continuous $\mathbb{R}^r$-valued functions with left-hand limits
$E\xi$	expectation of a random variable ξ
$\mathcal{F}$	σ-algebra
$\{\mathcal{F}_t\}$	filtration $\{\mathcal{F}_t, t \geq 0\}$
I_A	indicator function of a set A
I_n	$n \times n$ identity matrix
K	generic positive constant with convention $K + K = K$ and $KK = K$
$N(x)$	neighborhood of x
$O(y)$	function of y such that $\sup_y \lvert O(y)\rvert/\lvert y\rvert < \infty$
$O_1(y)$	function of y such that $\sup_y \lvert O(y)\rvert/\lvert y\rvert \leq 1$
$P(\xi \in \cdot)$	probability distribution of a random variable ξ
Q or Q^ε	generator of a Markov chain
$Q(t)$ or $Q^\varepsilon(t)$	generator of a Markov chain
$Qf(\cdot)(i)$	$= \sum_{j \neq i} q_{ij}(f(j) - f(i))$ where $Q = (q_{ij})$
$\mathbb{R}^r$	r-dimensional Euclidean space
a^+	$= \max\{a, 0\}$ for a real number a
a^-	$= \max\{-a, 0\}$ for a real number a
$a_1 \wedge \cdots \wedge a_l$	$= \min\{a_1, \ldots, a_l\}$ for $a_i \in \mathbb{R}$, $i = 1, \ldots, l$
$a_1 \vee \cdots \vee a_l$	$= \max\{a_1, \ldots, a_l\}$ for $a_i \in \mathbb{R}$, $i = 1, \ldots, l$
a.e.	almost everywhere
a.s.	almost surely
$\mathrm{diag}(A_1, \ldots, A_l)$	diagonal matrix of blocks $A_1, \ldots, A_l$
$\exp(Q)$	e^Q for any argument Q
$\log x$	natural logarithm of x
(l)	sequence $(l = 1, 2, \ldots)$ or a subsequence of it
$o(y)$	a function of y such that $\lim_{y \to 0} o(y)/\lvert y\rvert = 0$
$p^\varepsilon(t)$	$P(\alpha^\varepsilon(t) = 1, \ldots, \alpha^\varepsilon(t) = m)$ or $P(\alpha^\varepsilon(t) = 1, \alpha^\varepsilon(t) = 2, \ldots)$
w.p.1	with probability one
$(\Omega, \mathcal{F}, P)$	probability space
$\alpha(t)$ or $\alpha^\varepsilon(t)$	Markov chain with finite or countable state space
δ_{ij}	equals 1 if $i = j$ and 0 otherwise
ε	positive small parameter
ι	pure imaginary number with $\iota^2 = -1$
$\nu(t)$	quasi-stationary distribution
$\phi^{(n)}(t)$	sequence of functions (real or $\mathbb{R}^r$-valued)
$\sigma\{\alpha(s) : s \leq t\}$	σ-algebra generated by the process $\alpha(\cdot)$ up to t

$\mathbb{1}$	column vector with all components equal to one
$:=$	defined to be equal to
$\doteq$	approximately equal to
∇f	gradient of a function f
$\square$	end of a proof
$(a_1, \ldots, a_l) > 0$	$a_1 > 0, \ldots, a_l > 0$
$(a_1, \ldots, a_l) \geq 0$	$a_1 \geq 0, \ldots, a_l \geq 0$
$\lvert(a_1, \ldots, a_l)\rvert$	$= \sqrt{a_1^2 + \cdots + a_l^2}$
$\lvert y\rvert_T$	$= \max_{i,j} \sup_{0 \leq t \leq T} \lvert y_{ij}(t)\rvert$, where $y = (y_{ij}) \in \mathbf{R}^{r_1 \times r_2}$
$\langle a, b\rangle$	scalar product of vectors a and b
$\xi_n \Rightarrow \xi$	ξ_n converges to ξ weakly