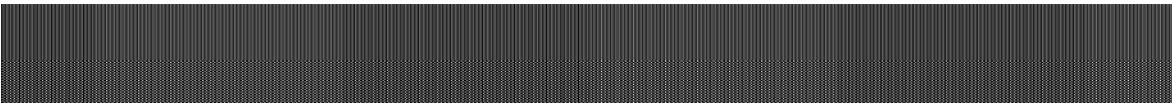


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# Robust Nonlinear Control Design

*State-Space and Lyapunov Techniques*

Randy A. Freeman  
Petar V. Kokotović

Reprint of the 1996 Edition

Birkhäuser  
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Randy A. Freeman  
Department of Electrical  
and Computer Engineering  
Northwestern University  
Evanston, IL 60208  
U.S.A.

Petar Kokotović  
Department of Electrical  
and Computer Engineering  
University of California  
Santa Barbara, CA 93106  
U.S.A.

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# Preface

This is the first book entirely dedicated to the design of robust nonlinear control systems. We believe that every effort in this direction is timely and will be highly rewarding in both theoretical and practical results.

Although the problem of achieving robustness with respect to disturbances and model uncertainty is as old as feedback control itself, effective systematic methods for the robust design of linear systems have been developed only recently. That such methods are already being successfully applied by a large community of practicing engineers testifies to a vital technological need.

Limitations of a popular methodology have always been among the factors stimulating new research. Such is the case with the inability of robust linear control to cope with nonlinear phenomena which become dominant when commands or disturbances cause the system to cover wide regions of its state space. In this situation it is natural to turn to nonlinear approaches to robust control design.

There are obvious reasons why robustness studies of nonlinear systems have been incomparably less numerous than their luckier linear cousins. The complexity of nonlinear phenomena is daunting even in the absence of disturbances and other uncertainties. It is not surprising that it has taken some time for a “clean” theory to discover classes of nonlinear systems with tractable analytic and geometric properties. During the last ten years, much progress has been made in this direction by nonlinear differential-geometric control theory. Most recently, a merger of this theory with classical Lyapunov stability theory led to the systematic adaptive “backstepping” design of nonlinear control systems with unknown constant parameters. However, the adaptive control paradigm is not suitable

for handling fast time-varying and functional uncertainties which are the main topic of this book.

Wide operating regimes involving large magnitudes of state and control variables, such as torques, pressures, velocities, and accelerations, are becoming increasingly common in modern aircraft, automotive systems, and industrial processes. In these regimes, nonlinearities which are not confined to “linear sectors” (namely those which exhibit super-linear growth) often cause severe, or even catastrophic, forms of instability. For this reason, our theory and design methods take such critical nonlinearities into account and focus on large-signal (global) behavior rather than small-signal (local) behavior. While not restricting nonlinear growth, we do consider systems with a particular structure.

Often a control design is performed on a model having no uncertainties. The robustness of the resulting system is then analyzed, possibly followed by a redesign to improve robustness. In contrast, our approach is to explicitly include uncertainties in the design model, taking them into account during the design itself. We therefore extend the theory behind Lyapunov design to include uncertainties by introducing the *robust control Lyapunov function* (rclf). Just as the existence of a control Lyapunov function is equivalent to the nonlinear stabilizability of systems without uncertainties, the existence of our rclf is equivalent to the nonlinear robust stabilizability of systems with uncertainties. The task of constructing an rclf thereby becomes a crucial step in robust nonlinear control design.

Our recursive methods for constructing rclf’s remove the “matching condition” constraint which severely limited the applicability of early robust Lyapunov designs. Already these designs exploited a worst-case differential game formulation, and we adopt a similar viewpoint in our approach to robust control design. Our solution of an inverse optimal robust stabilization problem shows that every rclf is the value function associated with a meaningful game. The resulting inverse optimal designs prevent the wasteful cancellation of nonlinearities which are beneficial in achieving the control objective, and they also inherit the desirable stability margins guaranteed by optimality.

The theoretical foundation of the entire book is established in Chapter 3 where we develop the rclf framework. Chapter 4 contains new results



in inverse optimality and relates them to crucial issues in control design and performance. The bulk of the design content of this book appears in Chapters 5–8. In Chapter 5 we present the recursive Lyapunov design procedure we call *robust backstepping*. This design procedure is modified to accommodate measurement disturbances in Chapter 6. A dynamic feedback version of backstepping is developed in Chapter 7. In Chapter 8 we combine these robust and dynamic backstepping methods to obtain a robust nonlinear version of classical proportional/integral (PI) control. Illustrative examples appear throughout the book, while Chapters 7 and 8 include detailed design examples.

This book is intended for graduate students and researchers in control theory, serving as both a summary of recent results and a source of new research problems. We assume the reader has a basic knowledge of nonlinear analysis and design tools, including Lyapunov stability theory, input/output linearization, and optimal control. For those readers not familiar with elementary concepts from set-valued analysis, we provide a review of set-valued maps in Chapter 2.

\*   \*   \*

We thank Tamer Başar for helping to direct our path, especially as we developed the inverse optimality results in Chapter 4. Also, we benefited greatly from frequent discussions with Miroslav Krstić and Ioannis Kanelakopoulos, whose contributions in adaptive nonlinear control directly inspired the dynamic backstepping methods in Chapters 7 and 8. We are grateful for the insights we gained from these colleagues. We thank Mohammed Dahleh, Laurent Praly, and Eduardo Sontag for sharing with us their technical expertise which helped shape many of our results. We are grateful to John Cheng of Rockwell International for providing us with physical examples motivating the material in Chapter 8. Many other researchers and educators influenced the content of this book, including Mrdjan Janković, Art Krener, Philippe Martin, Rodolphe Sepulchre, Stephen Simons, and Mark Spong.

Finally, this work would not have been possible without the patient support of our wives, Lisa and Anna—it is *analisa* that lies behind each of our control designs.

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Randy Freeman  
*Evanston, Illinois*

Petar Kokotović  
*Santa Barbara, California*

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