INTRODUCTION TO NONLINEAR AND GLOBAL OPTIMIZATION

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Aims and Scope

Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics, and other sciences.

The *Springer in Optimization and Its Applications* series publishes undergraduate and graduate textbooks, monographs, and state-of-the-art expository work that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.

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Preface

This book provides a solid introduction for anyone who wants to study the ideas, concepts, and algorithms behind nonlinear and global optimization. In our experience instructing the topic, we have encountered applications of optimization methods based on easily accessible Internet software. In our classes, we find that students are more often scanning the Internet for information on concepts and methodologies and therefore a good understanding of the concepts and keywords is already necessary.

Many good books exist for teaching optimization that focus on theoretical properties and guidance in proving mathematical relations. The current text adds illustrations and simple examples and exercises, enhancing the reader's understanding of concepts. In fact, to enrich our didactical methods, this book contains approximately 40 algorithms that are illustrated by 80 examples and 95 figures. Additional comprehension and study is encouraged with numerous exercises. Furthermore, rather than providing rigorous mathematical proofs, we hope to evoke a critical approach toward the use of optimization algorithms. As an alternative to focusing on the background ideas often furnished on the Internet, we would like students to study pure pseudocode from a critical and systematic perspective.

Interesting models from an optimization perspective come from biology, engineering, finance, chemistry, economics, etc. Modeling optimization problems depends largely on the discipline and on the mathematical modeling courses that can be found in many curricula. In Chapter 2 we use several cases from our own experience and try to accustom the student to using intuition on questions of multimodality such as "is it natural that a problem has several local optima?" Examples are given and exercises follow. No formal methodology is presented other than using intuition and analytic skills.

In our experience, we have observed the application of optimization methods with an enormous trust in clicking buttons and accepting outcomes. It is often thought that what comes out of a computer program must be true. To have any practical value, the outcomes should at least fulfill optimality conditions. Therefore in Chapter 3, we focus on the criteria of optimality illustrated with simple examples and referring further to the earlier mentioned books that present more mathematical rigor. Again, many exercises are provided.

The application and investigation of methods, with a nearly religious belief in concepts, like evolutionary programming and difference of convex programming, inspired us to explain such concepts briefly and then ask questions on the effectiveness and efficiency of these methods. Specifically, in Chapter 4 we pose questions and try to show how to investigate them in a systematic way. The style set in this chapter is then followed in subsequent chapters, where multiple algorithms are introduced and illustrated.

Books on nonlinear optimization often describe algorithms in a more or less explicit way discussing the ideas and their background. In Chapter 5, a uniform way of describing the algorithms can be found and each algorithm is illustrated with a simple numerical example. Methods cover one-dimensional optimization, derivative-free optimization, and methods for constrained and unconstrained optimization.

The ambition of global optimization algorithms is to find a global optimum point. Heuristic methods, as well as deterministic stochastic methods, often do not require or use specific characteristics of a problem to be solved. An interpretation of the so-called "no free lunch theorem" is that general-purpose methods habitually perform worse than dedicated algorithms that exploit the specific structure of the problem. Besides using heuristic methods, deterministic methods can be designed that give a guarantee to approach the optimum to an accuracy if structure information is available and used.

Many concepts exist which are popular in mathematical research on the structures of problems. For each structure at least one book exists and it was a challenge for us to describe these structures in a concise way. Chapter 6 explores deterministic global optimization algorithms. Each concept is introduced and illustrated with an example. Emphasis is also placed on how one can recognize structure when studying an optimization problem. The approach of branch and bound follows which aims to guarantee reaching a global solution while using the structure. Another approach that uses structure, the generation of cuts, is also illustrated. The main characteristic of deterministic methods is that no (pseudo-)random variable is used to find sample points. We start the chapter discussing heuristics that have this property. The main idea there is that function evaluations may be expensive. That means that it may require seconds, minutes, or even hours to find the objective function value of a suggested sample point.

Stochastic methods are extremely popular from an application perspective, as implementations of algorithms can be found easily. Although stochastic methods have been investigated thoroughly in the field of global optimization, one can observe a blind use of evolution-based concepts. Chapter 7 tries to summarize several concepts and to describe algorithms as basically and as dryly as possible, each illustrated. Focus is on a critical approach toward the results that can be obtained using algorithms by applying them to optimization problems.

We thank all the people who contributed to, commented on, and stimulated this work. The material was used and tested in master's and Ph.D. courses at the University of Almería where colleagues were very helpful in reading and commenting on the material. We thank the colleagues of the Computer Architecture Department and specifically its former director for helping to enhance the appearance of the book. Students and colleagues from Wageningen University and from Budapest University of Technology and Economics added useful commentary. Since 2008 the Spanish ministry of science has helped by funding a Ramón y Cajal contract at the Computer Architecture department of Málaga University. This enabled us to devote a lot of extra time to the book. The editorial department of Springer helped to shape the book and provided useful comments of the anonymous referees.

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