S*pringer* **M***onographs in* **M***athematics*

A. Nepomnyashchy I. Simanovskii
J.C. Legros

Interfacial Convection in Multilayer Systems

With 187 Illustrations

Springer

A. Nepomnyashchy
Department of Mathematics
Technion – Israel Institute of Technology
Haifa 32000
Israel
nepom@math.technion.ac.il

I. Simanovskii
Department of Mathematics
Technion – Israel Institute of Technology
Haifa 32000
Israel
yuri11@inter.net.il

J. C. Legros
Microgravity Research Centre
Universite Libre de Bruxelles, Belgium
50 av. F.D. Roosevelt
Bruxelles 1050
Belgium
jclegros@ulb.ac.be

Mathematics Subject Classification (2000): 76E17, 76E15

Library of Congress Control Number: 200692270

ISBN-10: 0-387-22194-8
ISBN-13: 978-0387-22194-6

Printed on acid-free paper.

© 2006 Springer Science+Business Media, LLC.
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC., 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed in the United States of America. (SPI/EB)

9 8 7 6 5 4 3 2 1

springer.com

Preface

The interfacial convection in multilayer systems is a widespread phenomenon that is of great importance in numerous branches of technology, including chemical engineering, space technologies, coating, etc.

The most well-known modern engineering technique that requires an investigation of the interfacial convection in systems with many interfaces is the liquid encapsulation crystal growth technique (Johnson, 1975; Géoris and Legros, 1996) used in space lab missions. It is known that time-dependent thermocapillary convection leads to solute segregation and, hence, to striation in crystals (Eyer *et al.*, 1985). The liquid encapsulation technique allows one to reduce significantly the convection and to grow high-quality, striation-free crystals by putting the melt between the fluid layers (Eyer and Leiste, 1985).

Another important problem that needs a multilayer approach for its self-consistent description is the coalescence between droplets or bubbles (Leshansky, 2001; Yeo *et al.*, 2001; Yeo and Matar, 2003) or between droplets and a bulk liquid (Savino *et al.*, 2003) under conditions of a heat/mass transfer. The influence of the interfacial convection on the coalescence rate, as well as the droplet migration under the action of an applied temperature/concentration gradient (Subramanian, 1981; Balasubramaniam and Subramanian, 2000), are significant for many engineering processes, including various extraction processes (Groothuis and Zuiderweg, 1960), steel refining (Mukai *et al.*, 2003), mixing (Fan and Zhang, 2001), and phase separation in alloys (Lu *et al.*, 2001).

One can mention also multilayer coating techniques used in the production of photographic films and multilayer fibers for optoelectronic devices (Yarin, 1995) and emulsified liquid membrane separation techniques widely used now in various extraction processes and industrial wastewater treatment (Noble and Douglas Way, 1987), where the interfacial convection appearing in a spherical liquid membrane layer can highly enhance mass transfer rates and sufficiently influence the stability of emulsified liquid membranes.

A scientific interest in such systems is due to the fact that the interfacial convection is characterized by a variety of physical mechanisms and types of instability, with the characteristic wave numbers ranging in a wide domain. The understanding of the underlying physical processes that can be achieved through the exploration of the mutual influence and the interaction between different interfaces is necessary for a successful application of this phenomenon. The problem of the convection in systems with many interfaces is important from the point of view of hydrodynamic stability theory, and of the theory of heat and mass transfer. Simultaneous interaction of interfaces with their bulk phases and with each other can lead to a much more complex dynamics and unexpected effects.

At present, the vast theoretical and experimental stuff related to the interfacial convection in multilayer systems is dispersed among numerous papers, many of which are hardly available. There are no monographs devoted to the interfacial convection in multilayer systems.

The present book, which contains a systematic investigation of convection in systems with interfaces, fills in this gap. For the first time, a classification of all known types of convective instability in such systems has been done, and the peculiarities of multilayer systems are discussed. The book provides an overview of the wide variety of steady and oscillatory patterns, waves, and other dynamic phenomena characteristic for multilayer fluid systems. Various physical effects, including heat and mass transfer, thermal and mechanical couplings on the interfaces, interfacial deformability, and the influence of surfactants on different types of convective motions are investigated using the multilayer approach; that is, the physical phenomena are studied in all fluids. This approach has its roots in the pioneering works of Sternling and Scriven (1959), Smith (1966), Zeren and Reynolds (1972), Imaishi and Fujinawa (1974a, 1974b), Gumerman and Homsy (1974a, 1974b), Gershuni and Zhukhovitsky (1982), Ferm and Wollkind (1982), Renardy and Joseph (1985), Renardy (1986), and Wahal and Bose (1988).

The book consists of seven chapters. In Chapter 1 (Introduction), the main mathematical models, which are used in subsequent parts of the book, are formulated. Chapter 2 contains the description of numerous types of convective instability in different systems with a single interface. In addition to the well-known monotonic Rayleigh-Bénard and Bénard-Marangoni instabilities, the systems with an interface are subject to many kinds of oscillatory instability generated by buoyancy, by the thermocapillary effect, by the competition between buoyancy and thermocapillarity, by the mode mixing of interfacial and internal waves, by an interplay between a thermal gradient and a surfactant distribution, and so on. Also, a specific non-Rayleigh mechanism of instability can produce a buoyancy instability by heating from above (anticonvection). All of these phenomena are studied both in the framework of a linear instability theory and by a nonlinear analysis.

The manifestations of the interfacial convective instabilities in multilayer systems are studied in Chapters 3 and 4. In Chapter 3, the situation in which the deformations of interfaces are not significant is considered. The main part of this chapter is devoted to the investigation of the Marangoni convection in three-layer systems, which is studied theoretically and by means of experiments (on the Earth and in space). Also, the peculiarities of the Rayleigh convection, mixed Rayleigh-Marangoni convection, and anticonvection are described. Situations that are impossible in the case of a single interface are analyzed.

The types of instability, which are essentially connected with interfacial deformations, are explored in Chapter 4. The progress in understanding of the nonlinear aspects of the problem is achieved by means of the long-wave asymptotic approach.

Chapter 5 presents the results on the stability of convective flows in systems with interfaces. The problems studied in this chapter include different types of instability of thermocapillary and buoyancy-thermocapillary flows, with and without interfacial deformations.

A specific class of interfacial phenomena related to the dynamics of very thin films is considered in Chapter 6. Chapter 7 is devoted to the discussion of the new directions for the scientific analysis and applications.

In our opinion, this book will be useful for experts in fluid mechanics, heat and mass transfer theory, nonlinear dynamics, and applied mathematics as well as for physicists and chemical engineers interested in the investigation of the interfacial physico-chemical processes and in their applications. The book can be used also by graduate students.

The scientific results included into this book have been obtained in collaboration with our co-authors and friends: Th. Boeck, L. M. Braverman, K. Eckert, Ph. Géoris, A. Yu. Gilev, A. A. Golovin, M. Hennenberg, I. L. Kliakhandler, A. Oron, A. Thess, S. Van Vaerenbergh, A. Viviani, I. I. Wertgeim, and M. A. Zaks. This book could not be written without fruitful discussions and scientific advice of our colleagues: I. Aranson, G. I. Burde, P. Colinet, S. H. Davis, C. S. Iorio, E. Istasse, D. Kessler, G. Lebon, B. Matkovsky, D. Melnikov, R. Narayanan, L. M. Pismen, A. Ye. Rednikov, B. Scheid, D. Schwabe, V. Shevtsova, L. Shtilman, G. Sivashinsky, and M. G. Velarde.

We dedicate this book to the memory of our late teachers and friends E. M. Zhukhovitsky and G. Z. Gershuni.

Our work on the subject of the book was sponsored by grants of the German-Israeli Foundation and of the Israel Science Foundation. The preparation of the book has been carried out in the framework of the ICOPAC (Interfacial Convection and Phase Change) network sponsored by the European Union.

Contents