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350

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Global Optimization



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To our families

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Preface

The continuous, essentially unconstrained optimization problem is addressed.

For many practical optimization problems the unimodality cannot be proved. On the contrary, rather often these problems have proved multimodal. Unsophisticated use of local optimization techniques is normally inefficient for solving such problems. Therefore, more sophisticated methods designed for global optimization are important from a practical point of view.

This need has been widely recognized and a big variety of global optimization methods has been designed. However, it is not clear which problems some specific method is applicable to. From the separate papers dealing with specific algorithms it is difficult to draw conclusions which would make it possible to clarify this point.

One way to overcome this problem is to bring together all approaches in a single treatise. The first attempt in this direction was the publication of the two volumes named *Towards Global Optimization* [Dixon and Szegö 1975, 1978], which contained collections of early papers on global optimization. These books very much accelerated the research activity in the field.

Now a decade later the field has reached some maturity and we think that a more uniform exposition than mere collected papers could better contribute to the future development of global optimization theory and its application.

The book is written to stimulate the research in the field, and to promote the use of global optimization methods by giving an introduction to different approaches.

It is assumed that the readers are familiar with basic concepts of mathematics and computing. Also general concepts of continuous local optimization are assumed to be known by the reader.

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Aimo Törn, Antanas Žilinskas

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