Braumann, C. A. (2019). Introduction to Stochastic Differential Equations with Applications to Biology and Finance. Wiley, Hoboken NJ, 304 pages. ISBN: 978-1-119-16606-1, 978-1-119-16608-5 (ePub), 978-1-119-16607-8 (Adobe PDF).

NOTE: BOOK EXERCISE SOLUTIONS available on the companion website:

https://media.wiley.com/product_ancillary/63/11191660/download/solutions.pdf

TABLE OF CONTENTS

Preface xi

About the companion website xv

1 Introduction 1

- 2 Revision of probability and stochastic processes 9
- 2.1 Revision of probabilistic concepts 9
- 2.2 Monte Carlo simulation of random variables 25
- 2.3 Conditional expectations, conditional probabilities, and independence 29
- 2.4 A brief review of stochastic processes 35
- 2.5 A brief review of stationary processes 40
- 2.6 Filtrations, martingales, and Markov times 41
- 2.7 Markov processes 45

3 An informal introduction to stochastic differential equations 51

4 The Wiener process 57

- 4.1 Definition 57
- 4.2 Main properties 59
- 4.3 Some analytical properties 62
- 4.4 First passage times 64
- 4.5 Multidimensional Wiener processes 66

5 Diffusion processes 67

- 5.1 Definition 67
- 5.2 Kolmogorov equations 69
- 5.3 Multidimensional case 73

6 Stochastic integrals 75

- 6.1 Informal definition of the Itô and Stratonovich integrals 75
- 6.2 Construction of the Itô integral 79
- 6.3 Study of the integral as a function of the upper limit of integration 88
- 6.4 Extension of the Itô integral 91
- 6.5 Itô theorem and Itô formula 94
- 6.6 The calculi of Itô and Stratonovich 100
- 6.7 The multidimensional integral 104

7 Stochastic differential equations 107

- 7.1 Existence and uniqueness theorem and main proprieties of the solution 107
- 7.2 Proof of the existence and uniqueness theorem 111
- 7.3 Observations and extensions to the existence and uniqueness theorem 118

8 Study of geometric Brownian motion (the stochastic Malthusian model or Black– Scholes model) 123

- 8.1 Study using Itô calculus 123
- 8.2 Study using Stratonovich calculus 132

9 The issue of the Itô and Stratonovich calculi 135

- 9.1 Controversy 135
- 9.2 Resolution of the controversy for the particular model 137
- 9.3 Resolution of the controversy for general autonomous models 139

10 Study of some functionals 143

- 10.1 Dynkin's formula 143
- 10.2 Feynman-Kac formula 146

11 Introduction to the study of unidimensional Itô diffusions 149

- 11.1 The Ornstein–Uhlenbeck process and the Vasicek model 149
- 11.2 First exit time from an interval 153
- 11.3 Boundary behaviour of Itô diffusions, stationary densities, and first passage times 160

12 Some biological and financial applications 169

- 12.1 The Vasicek model and some applications 169
- 12.2 Monte Carlo simulation, estimation and prediction issues 172
- 12.3 Some applications in population dynamics 179
- 12.4 Some applications in fisheries 192
- 12.5 An application in human mortality rates 201

13 Girsanov's theorem 209

- 13.1 Introduction through an example 209
- 13.2 Girsanov's theorem 213

14 Options and the Black–Scholes formula 219

- 14.1 Introduction 219
- 14.2 The Black–Scholes formula and hedging strategy 226
- 14.3 A numerical example and the Greeks 231
- 14.4 The Black–Scholes formula via Girsanov's theorem 236
- 14.5 Binomial model 241
- 14.6 European put options 248
- 14.7 American options 251
- 14.8 Other models 253

15 Synthesis 259

References 269

Index 277